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POWER SERIES SOLUTION OF DEFLECTION OF BEAMS ON NONLINEAR FOUNDATION

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DEDICATION

To my parents,

To Maha & Sara

ACKNOWLEDGMENT

I would like to express my deepest gratitude to Dr. Mazen Al-Qaisi for his valuable cooperation and advice throughout all stages of this research work.

Also I express my great thanks to my brother Eng. Nayef Al-Husban & Mr. Ibrahim Al-Hawamdeh for their help throughout my degree study.

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NOMENCLATURE

P(n)	The Load coefficients in the form of power series.
A(n)	The Deflection coefficients of power series .
n	The Power series degree.
AA(n)	The Non linear coefficients .
P(x)	The non dimensional Load applied transversely on the beam.
P1(x)	The Load applied transversely on the beam(N).
x	The Distance on the beam beam axis(m).
I	Second moment of area of the cross-section (m^4).
E	Young's modules of beam elasticity (N/m^2).
m	The Largest degree of power series .
M	Bending moment (N/m^2).
C	Distance from center to top of the beam cross section (m).
U_B	The Strain energy due to bending (N.m).
U_{EF}	The Strain energy due to elastic foundation(N.m).
U_P	The Load potential (N.m).
U_S	The Strain energy due to stretching load(N.m).
L	The Length of the beam(m).
$q(w)$	Force per unit deflection of the elastic foundation(N/m).

R_1	The Radius of curvature before deflection(m).
R_2	The Radius of curvature after deflection(m).
p_1	The Undeflected point on the beam.
p_2	The Deflected point of p_1.
u	The Axial displacement(m).
V	The Total potential energy(N.m).
$\bar{V}$	The Total potential energy functional (N.m).
dx	The Straight element of undeflected beam(m).
ds	The Curved element of deflected beam(m).
Tol	The Tolerance (10.0 E-4)
$W(x)$	The Beam deflection at point x
$W'(x)$	The Slope of the beam deflection at point x
$W''(x)$	The 2nd derivative of the beam deflection at point x
$W'''(x)$	The 3rd derivative of the beam deflection at point x
$W''''(x)$	The 4th derivative of the beam deflection at point x
W_P	The linear particular solution
W_h	The linear homogenous
END	The End of the beam (x=1.0)

Greek Symbols :

$\alpha 1$	The Linear stiffness of foundation (N/ m^2).
$\beta 1$	The Non linear stiffness of foundation(N/ m^4).
α	The non dimensional Linear stiffness of foundation

β	The non dimensional Non linear stiffness of foundation
$\sum_{n}^{m}$	The Summation symbol from n to m
σ_x	The Maximum normal stress(N/ m^2).
χ	The Change in curvature (m^{-1})
ψ'	The Rate of change of slope.
ζ	The Non dimensional coordinate for beam.
ψ	The Slope of deflected beam .
δV	The First variation of the functional.

ABSTRACT

POWER SERIES SOLUTION OF DEFLECTION OF BEAMS ON NONLINEAR FOUNDATION

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Deflections of elastic beams resting on nonlinear foundation under static loads are normally analyzed approximately by several methods, such as perturbation, our study which is the first attempt to solve this problem by power series method which has no restriction on the order of non linearity.

The governing differential equation of deflections of elastic beam has the form of four order and depends on many parameters which are the foundation stiffness coefficients (linear & non linear), modulus of elasticity of beam material, moment of inertia of cross section of the beam and the shape of the external load which is transversely applied to beam.

We assumed that the deflection has the form of power series, two kinds of beam are studied - clamped and simply supported - under three different types of static loads; constant load, unsymmetric cubic load and symmetric sinusoidal load, then we solved the differential equation for each type under certain boundary conditions at both ends of the beam.

First approximation of solution is taken from the linear solution of the problem, then searching is made for nonlinear deflections under the condition of satisfying the boundary conditions of nonlinear problem to achieve the exact solution within the given tolerance.

Convergence of power series method was examined to determine the best value of its degree at midspan of the beam.The effect of foundation coefficients were analyzed throughout the solutions, also the effect of the shape of static loads and the symmetry were studied, deflections of each case were plotted versus the beam axis.

Bending stress due to each load / case was calculated and then plotted graphically for different values of foundation coefficients.

Computer programs in FORTRAN are set to solve the problem. Harmonic balance method for simply supported beams was used to make comparison with power series method, the trend for both methods was found identical.

The power series method was found to be a very powerful method when computerized.

CHAPTER ONE

INTRODUCTION

1-1 Introduction

Almost all engineering materials possess to a certain extent the property of elasticity, if the external loads producing deformation do not exceed a certain limit, the deformation some times goes in linear shape other times goes in non linear shapes and this comes from nonlinear foundation.

We assume that the material is homogeneous and isotropic (i.e. elastic properties are the same in all directions), the loads are static, and the deflections are within the elastic limit but nonlinear.

Calculations of deformations, strains and their stresses, then their maximum values are very important in design purposes.

1-2 Differential Equation :

The governing differential equation for small deflections on nonlinear foundation of beams is one of the most important equations since it appears in many physical and engineering problems.

The general form of this equation is given as

$$EI\, w''''(x) + \alpha 1\, w(x) + \beta 1\, w^3(x) = P(x) \qquad (1\text{-}1)$$

where $W(x)$ is beam deflection

$\alpha 1$ is linear stiffness of foundation

$\beta 1$ is non linear stiffness of foundation

I : Second moment, moment of inertia

E : modulus of elasticity of beam material (Young's modulus)

P(x): is external distributed load applied transversely on beam, which could be constant polynomial or sinusoidal or any function on the form of power series.

All derivatives in equation (1-1) are with respect to x (beam axis).The values of linear and nonlinear stiffness of foundation are not necessarily small.

Equation (1.1) is fourth order non-linear differential equation which has no solution, many approximate analytical methods such as harmonic balance and energy ... etc. or numerical methods or even experimental methods may be used to solve this equation.

This is the first attempt to solve the equation by a power series method.

1-3 Solution Description of Differential equation :

The solution of equation (1.1) describes the deflection shape of the beam at any point, this deflection is influenced by each term in the equation, The situation of the beam specifies boundary conditions which apply at the ends of the beam, in our study we investigate two kinds of beams clamped beams where both the deflection and slope at the ends (x= 0 & x=1.0) must vanish.

The second kind is simply supported beams where the deflection and moment at both ends are zero .

Among the variety of available descriptors of the solution of 4th order non linear differential equation, the following are of great significant on the beam deflection.

1- Kind of beam

a - clamped & b- simply supported

2- Type of load

a- constant load

b- cubic polynomial load (unsymmetric)

c- Sinusoidal load (symmetric)

3- Linear stiffness ($\alpha 1$)

4- Non linear stiffness ($\beta 1$)

For different values of $\alpha 1$ & $\beta 1$ at each type of load for clamped & simply supported beams, Deflection & bending stresses along the beam are plotted.

Convergence of power series of degree n is plotted versus the deflection and bending stress of the beam at midspan (x=0.5) to determine the best value of n .

CHAPTER TWO

LITERATURE SURVEY

The property of small deflections on nonlinear foundation of beam that does not exceed the elastic limits of the material is of both analytical and technological interest, it is indispensable to achieve high accuracy and reliability of the mechanical components.

In certain applications, a beam of relatively small bending stiffness is placed on an elastic foundation, loads are transversely applied to the beam, the loads are transferred through the beam to the foundation, the beam and foundation must be designed to resist the loads without failing.

Normally, failure occurs in the beam before it occurs in the foundation. So, we assume in this thesis that the foundation has sufficient strength to resist the applied loads.

Furthermore, we assume that the foundation resists the loads transmitted by beam in a non-linearly elastic manner, that is the stress developed at any point between the beam and the foundation is nonlinearly proportional to the deflection of the beam at that point.

In spite of the fact that nonlinear deflection of beams is very important in many applications, few researchers dealt with this problem .

Chueh eepsakul, S.Bancharon, S.Wang , C.M. (1994), analyzed deflection of beams under moment gradient, one end of the beam is hinged, the other end slides freely over frictionless support, Differential equation is solved by using elliptic integral method which yields a closed-form solution, then they solved the governing differential equation numerically by the shooting-optimization technique (fourth order Runge-Kutta-algorithm), comparison studies of the results obtained from the two methods are made, the results found to be in very good agreement.

Ohtsuki, Atsumi Yasui and Tauyoshi (1994), have investigated non linear deflections in unmovable simply supported flexible beam subjected to bending moment by using test method experimentally. Experimental data agree very closely with theoretical results, which helps the analysis and design of flexible components.

Hillsman, Veranons S.Tomovic, Mileta M. (1995) determined beam deflections at any point along the length of a beam by using the double integration method , used computer algebra software to help automate the beam deflection process moment equation is developed, software is used to solve the constants of integration, then final deflection equation is developed, then deflection curves are plotted .

Disciuva, Marco Icard: Vgo (1995) have used Euler method of the adjacent equilibrium configurations to solve the Von Karman nonlinear strain displacement relations of antisotropic, laminated Timosherko beams. Numerical results is concluded that the effectiveness of the control depends on the boundary conditions, mechanisms of activation and lay-ups.

Ohtusk:, Astsumi Tsurumi, Tohra (1996) observed a large deflection characteristics of cantilever beams (free edge) without exceeding the elastic limit of the materials, they analyzed the deflection of the beam with low support stiffness experimentally. Displacement and bending stresses are examined under a load applied at the free end to achieve high reliability of the structural components as aircraft wings.

In our study the power series method is used to solve the non linear fourth order differential equation to get the deflections of clamped and simply supported beams on nonlinear foundation under static transversal loads in different types like sinusoidal loads, cubic loads and constant loads.

The effect of varying the parameters of linear and nonlinear stiffness of foundation with satisfying the beam ends boundary conditions will be studied in this work .

By considering the work done in literature , we note that there are no work available dealing with solving the small deflections on nonlinear foundation by power series method .

CHAPTER THREE

FORMULATION OF THE NON-LINEAR DIFFERENTIAL EQUATION FOR DEFLECTIONS OF BEAMS ON NONLINEAR FOUNDATION

3-1 Introduction

The response of a beam resting on an elastic foundation to loads is described by a single differential equation subjected to different boundary conditions for the beams depending on how the beam is supported at its ends. Loads in general, cause the beam to deflect, which in turn displaces the elastic foundation, establishing the energy or work functional of all forces in the problem leads formulating the equation by minimizing the functional .

3-2 Establishing the potential energy functional of the problem

Consider a clamped beam resting on an elastic foundation with linear stiffness $\alpha 1$ & nonlinear stiffness $\beta 1$, under a uniformly distributed load P(x) on the beam.

Neglecting the stretching energy of the beam (axial extensibility) as well as shear deformation energy. This neglecting is an engineering approximation. based on the fact that energy due to axial strain and shear deformation is very small.

The only strain energy left are both the strain energy of bending and the strain energy due to the elastic foundation, the expression for that is :

$$U_B + U_{EF} = \int_0^1 \frac{1}{2} EI\chi^2 dx + \int_0^L [\int_0^w q(W) dW] dx \tag{3-1}$$

where

U_B : The strain energy due to the binding .

U_{EF} : The strain energy due to the elastic foundation .

χ : The change in curvature .

E : Young's modulus .

I : Second moment of area of the cross section.

L : The length of the beam

$q(W)$: The force per unit deflection of the elastic foundation, and is given as

$$q(W) = \alpha 1 W + \beta 1 W^3 \tag{3-2}$$

where

$\alpha 1$: linear foundation constant.

$\beta 1$: non linear foundation constant.

So the strain energy due to the elastic foundation becomes :

$$U_{EF} = \int_0^L \int_0^w [\alpha 1 W + \beta 1 W^3] dW dx$$

$$= \int_0^L (\frac{1}{2}\alpha 1W^2 + \frac{1}{4}\beta 1W^4)dx \qquad (3\text{-}3)$$

substituting equation (3-3) into equation (3-1) yields

$U = U_B + U_{EF}$

$$= \int_0^L \frac{1}{2} EI\chi^2 dx + \int_0^L (\frac{1}{2}\alpha 1W^2 + \frac{1}{4}\beta 1W^4)dx \qquad (3\text{-}4)$$

The load potential, on the other hand, is equal to the load times the deflection of the beam as shown in Fig (3-1)

$$U_p = PW \qquad (3\text{-}5)$$

Where

U_p : The load potential

P : Transverse load (applied load P(x))

W : The deflection of the beam.

So the total potential energy function is :

$$V = \int_0^L \frac{1}{2} EI\chi^2 dx + \int_0^L (\frac{1}{2}\alpha 1W^2 + \frac{1}{4}\beta 1W^4)dx - P1W \qquad (3\text{-}6)$$

Now we need to express χ in terms of the displacement vector, i.e. to find the change of curvature χ in terms of the displacement vector . Referring to the geometry of the problem and from Figure (3-1) if we select point p_1 as the undeflected point on the beam, and p_2 its new state, deflected point, so the displacement vector taking p_1,p_2 is thought to be decomposed into two components, the axial displacement component u and the lateral component W which is perpendicular to u.

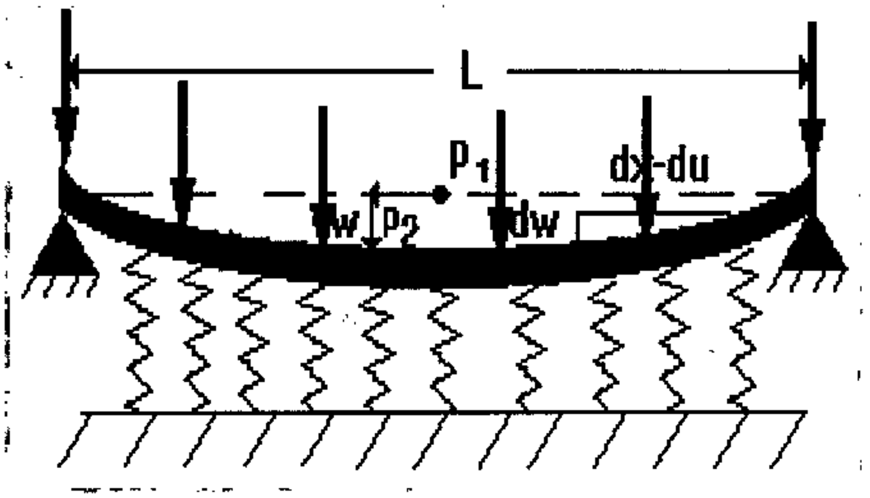


Figure (3-1) Deflection of simply supported beam on non linear elastic foundation

Here we are assuming the central line of the beam to be inextensible, so that the length L of the beam before deflection is also L after deflection, consequently, a straight element of the undeflected beam dx and a curved element of a deflected beam ds are equal (dx = ds) the slope at any point of the deflected beam is w Fig (3-2)shows small element of deflected beam enlarged.

From differential geometry, we know that this element is equivalent to that of Fig (3-3) since the difference between the curved and straight triangle vanishes when taking the differential limit .

Note also that horizontal projection of ds=dx is dx-du. This is because the horizontal length shrank due to the u displacement component.

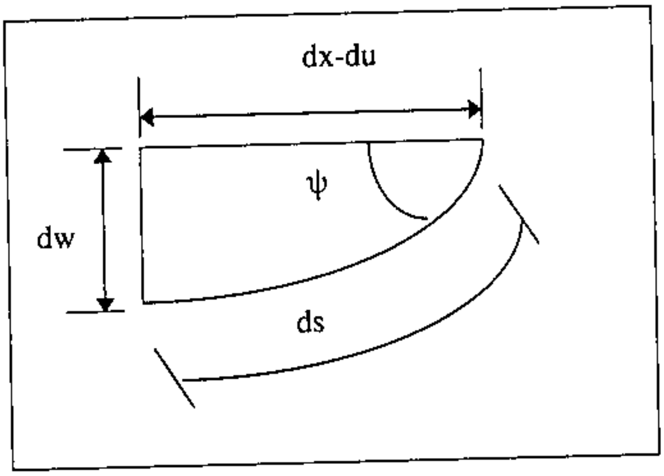


Figure (3-2) :Exact deflection of the beam

In order to find the curvature of the deflected beam in terms of W, we examine Fig. (3-3) for which it is evident that :

$$\sin(\psi) = \frac{dW}{ds} = \frac{dW}{dx} = W' \tag{3-7}$$

thus $\psi = \sin^{-1}(W')$

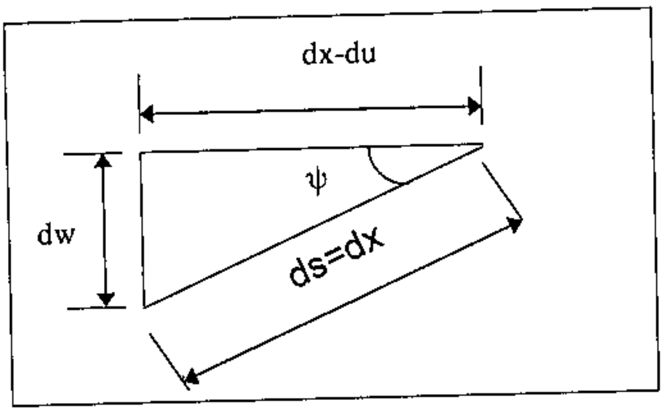


Figure (3-3) : Approximate deflection of beam since the curvature is the rate of change of the slope ψ' with x we find that :

$$\frac{1}{R_2} = \psi' = (\sin^{-1}(W'))' = \frac{W''}{(1-W'^2)^{\frac{1}{2}}} \qquad (3\text{-}8)$$

Noting that the curvature of the beam before deflection was :

$$\frac{1}{R_1} = \frac{1}{\psi} = 0 \qquad (3\text{-}9a.)$$

we see that the change in curvature is

$$\chi = \frac{1}{R_2} - \frac{1}{R_1} = \frac{W''}{(1-W')^{\frac{1}{2}}} \qquad (3\text{-}9b.)$$

the preceding derivatives were purely differential geometrical ones, and did not use any mechanical principles except perhaps the concept a point moving from p1 to p_2 which is strictly speaking a kinematical problem .

Now we come to the physical and mechanical considerations, the total potential energy of the beam is defined as the difference between the total strain energy and the load potential .

V= Potential energy due to bending + Strain energy due to foundation + Energy due to stretching - Load potential.

$$= U_B + U_{EF} + U_S - U_P \qquad (3\text{-}12)$$

$$= U - U_P.$$

where can be expressed as :

$$U = \int_0^L \frac{1}{2} EA\xi^2 dx + \int_0^L \frac{1}{2} EI\chi^2 dx + \int_0^L (\frac{1}{2}\alpha 1W^2 + \frac{1}{4}\beta 1W^4)dx \quad (3\text{-}13a.)$$

As we have assumed the central line to be inextensible ds=dx the axial strain is zero and so is the stretching energy U_S , thus the total potential is

$$V = \int_0^L \frac{1}{2} EI\chi^2 + \int_0^L (\frac{1}{2}\alpha 1W^2 + \frac{1}{4}\beta 1W^4)dx - P1W . \quad (3\text{-}13b)$$

Inserting the expressions found for χ in terms of w (from equation (3-9) & (3-11)) in V we obtain .

$$V = \int_0^L [\frac{1}{2} EI \frac{W''^2}{(1 - W'^2)} + \frac{1}{2}\alpha 1W^2 + \frac{1}{4}\beta 1W^4 - P1W]dx \quad (3\text{-}14)$$

Now expand the term $(1 - w'^2)^{-1}$ in the power series we get .

$$(1 - W'^2)^{-1} = 1 + W'^2 + W'^4 + ... \quad (3\text{-}15)$$

substituting the expended form into equation (3-14) yields

$$V = \int_0^L \left\{\frac{1}{2} EIW''^2 (1 + W'^2 + W'^4 + ...) + \frac{1}{2}\alpha 1W^2 + \frac{1}{4}\beta 1W^4 - P1W\right\}dx \quad (3\text{-}16)$$

where we have written explicitly terms up to the fourth order.

Taking the first two term approximation of expanding series and assuming the rest are very small we get

$$V = \int_0^L \left\{\frac{1}{2} EIW''^2 + \frac{1}{2} EIW''^2 W'^2 + \frac{1}{2}\alpha 1W^2 + \frac{1}{4}\beta 1W^4 - P1W\right\}dx \quad (3\text{-}17)$$

also assuming $\frac{1}{2} EW''^2 W'^2$ is very small.

$$V = \int_0^L \left\{\frac{1}{2} EIW''^2 + \frac{1}{2}\alpha 1W^2 + \frac{1}{4}\beta 1W^4 - P1W\right\}dx \quad (3\text{-}18)$$

Consider the general case when the potential energy functional is a function of W'',W' as well as W we seek to find the first variation of this function which means the total potential energy must be stationary and its first derivative must vanish.

so

$$\delta V = \delta\int_0^L \overline{V}(W'',W',W,x)dx$$

$$= \int_0^L \delta\overline{V}dx$$

$$= \int\left(\frac{\partial\overline{V}}{\partial W''}\delta W'' + \frac{\partial\overline{V}}{\partial W'}\delta W' + \frac{\partial\overline{V}}{\partial W}\delta W\right)dx \qquad (3\text{-}19a.)$$

where

$$\overline{V} = \left(\frac{1}{2}EIW''^2 + \frac{1}{2}\alpha 1W^2 + \frac{1}{4}\beta 1W^4 - PW\right) \qquad (3\text{-}19b.)$$

The terms with $\delta W''$ and $\delta W'$ must be rewritten using integration by parts into the form including only δW

$$\int_0^L \frac{\partial\overline{V}}{\partial W''}\delta W''dx = \frac{\partial\overline{V}}{\partial W''}\delta W'\Big|_0^L - \int_0^L\left(\frac{\partial\overline{V}}{\partial w''}\right)'\delta W'dx$$

$$= \frac{\partial\overline{V}}{\partial W''}\delta W'\Big|_0^L - \left(\frac{\partial\overline{V}}{\partial W''}\right)'\delta W\Big|_0^L + \left(\frac{\partial\overline{V}}{\partial W''}\right)''\delta Wdx \qquad (3\text{-}20a.)$$

$$\int_0^L \frac{\partial\overline{V}}{\partial W'}\delta W'dx = \frac{\partial\overline{V}}{\partial W'}\delta W\Big|_0^L - \int_0^L\left(\frac{\partial\overline{V}}{\partial W'}\right)'\delta Wdx \qquad (3\text{-}20b.)$$

substitute (3-20 a & b) into equation(3-19a.) you will get :

$$\delta V = \int_0^L \left[\left(\frac{\partial \bar{V}}{\partial W''} \right)'' - \left(\frac{\partial \bar{V}}{\partial W'} \right)' + \left(\frac{\partial \bar{V}}{\partial W} \right) \right] \delta W dx +$$

$$\left(\frac{\partial \bar{V}}{\partial W''} \right) \delta W' \Big|_0^L + \left(\frac{\partial \bar{V}}{\partial W'} \right) \delta W \Big|_0^L - \left(\frac{\partial \bar{V}}{\partial W''} \right)' \delta W \Big|_0^L \qquad (3\text{-}21)$$

Following the principle of the stationary value of the total potential energy, the equilibrium condition of the beam is marked by the vanishing of the first variation of the total potential energy i.e. by

$$\delta V = 0 \qquad (3\text{-}22)$$

and since δW is arbitrarily small but non zero, we get the differential equation

$$\left(\frac{\partial \bar{V}}{\partial W''} \right)'' - \left(\frac{\partial \bar{V}}{\partial W'} \right) + \left(\frac{\partial \bar{V}}{\partial W} \right) = 0 \qquad (3\text{-}23)$$

we may also write the terms of boundary condition as

$$\left(\frac{\partial \bar{V}}{\partial W''} \right) \delta W' \Big|_0^L + \left(\frac{\partial \bar{V}}{\partial W'} \right) \delta W \Big|_0^L - \left(\frac{\partial \bar{V}}{\partial W''} \right)' \delta W \Big|_0^L = 0 \qquad (3\text{-}24)$$

where

$$\left(\frac{\partial \bar{V}}{\partial W} \right) = (-P1 + \alpha 1 W + \beta 1 W^3) \qquad (3\text{-}25a.)$$

$$\left.\begin{aligned}
&\left(\frac{\partial \bar{V}}{\partial W''}\right) = EIW'' \\
&\left(\frac{\partial \bar{V}}{\partial W''}\right)' = EIW''' \\
&\left(\frac{\partial \bar{V}}{\partial W''}\right)'' = EIW'''' \\
&\left(\frac{\partial \bar{V}}{\partial W'}\right) = 0 \\
&\left(\frac{\partial \bar{V}}{\partial W'}\right)' = 0
\end{aligned}\right\} \qquad (3\text{-}25b.)$$

Also for clarity we may write the terms of boundary conditions of equation (3-24) explicitly as

$$\left(EIW''(L)\delta W'(L) - EIW''(0)\delta W'(0)\right) - \left(EIW'''(L)\delta W(L) - EIW(0)\delta W(0)\right) = 0 \qquad (3\text{-}26)$$

Substituting equation (3-25 a,b) into equation (3-23) to find the general form of the governing fourth order non linear differential equation of a beam on elastic foundation

$$\text{EI } W'''' + \alpha 1\, W + \beta 1\, W^3 = \text{P1} \qquad (3\text{-}27)$$

we may look at the different ways of supporting the beam and then see how we apply boundary conditions in solving the above equation.

3-3 Reduction of the governing differential equation to the non-dimensional form.

The next setp is to write the differential equation in non-dimensional form. If the non-dimensional position η is defined such that:

$$\eta = \frac{x}{L} \tag{3-28}$$

and from this definition, it follows that

$$\frac{d\eta}{dx} = \frac{1}{L} \tag{3-29}$$

Using a chain rule for differentiation, with the help of the equation (2-29)

$$\frac{dW}{dx} = \frac{dW}{d\eta}\frac{d\eta}{dx} = \frac{1}{L}\frac{dW}{d\eta} \tag{3-30a}$$

$$\frac{d^2W}{dx^2} = \frac{d}{d\eta}(\frac{dW}{dx})\frac{d\eta}{dx} = \frac{1}{L^2}\frac{d^2W}{d\eta^2} \tag{3-30b}$$

$$\frac{d^3W}{dx^3} = \frac{d}{d\eta}(\frac{d^2W}{dx^2})\frac{d\eta}{dx} = \frac{1}{L^3}\frac{d^3W}{d\eta^3} \tag{3-30c}$$

$$\frac{d^4W}{dx^4} = \frac{d}{d\eta}(\frac{d^3W}{dx^3})\frac{d\eta}{dx} = \frac{1}{L^4}\frac{d^4W}{d\eta^4} \tag{3-30d}$$

Substituting equations (3-30) into the differential equation (3-27), one can get :

$$\mathrm{EI}\ \frac{1}{L^4}\frac{d^4W}{d\eta^4} + \alpha 1\, W + \beta 1\, W^3 = \mathrm{P1} \tag{3-31}$$

Devide all terms of the previous differential equation by the non-zero term $\frac{EI}{L^4}$, yields

$$\frac{d^4W}{d\eta^4}+\alpha 1\frac{L^4}{EI}W+\beta 1\frac{L^4}{EI}W^3=\mathrm{P}1\frac{L^4}{EI} \qquad (3\text{-}32)$$

Assume the following non-dimensionalization terms :

$$\left.\begin{aligned} P&=\frac{P1L^2}{EI}\\ \alpha&=\frac{\alpha 1L^4}{EI}\\ \beta&=\frac{\beta 1L^4}{EI}\end{aligned}\right\} \qquad (3\text{-}33)$$

The differential equation of its non-dimensional form takes its final shape as:

$$\frac{d^4W}{d\eta^4}+\alpha W+\beta W^3=P \qquad (3\text{-}34)$$

CHAPTER FOUR

SOLUTION PROCEDURE.

4-1 Introduction

Solution of fourth order non-linear differential equation for large deflections of beams by analytical methods is not easy especially by power series method which will have a number of unknowns (power series coefficients) equal to the power series exponent n.

The non linear term should be reduced as the same order of w, (*see Appendix C*).To get first feeling of solution the equation should be solved as linear differential equation which means α & β are zero.

The beam is taken as unity, and divided into the 10 equal elements, deflection and stress were calculated at both ends of each element .

4-2 Problem formulation

Starting with fourth order nonlinear differential equation 1-1

$$W''''(x) + \alpha W(x) + \beta W^3(x) = P(x) \qquad (4\text{-}1)$$

Assuming the deflection in the form of the power series

$$W(x) = \sum_{n=0}^{m} A_n x^n$$

$$= A_0 + A_1 x + A_2 x^2 + A_3 x^3 + \dots + A_m x^m$$

$$= A(0) + A(1)\,x + A(2)\,x^2 + A(3)\,x^3 + \dots + A(m)x^m \qquad (4\text{-}2)$$

m will be determined from convergence plot.

$$W'(x) = \sum_{n=1}^{m} n\,A_n\,x^{n-1} \qquad (4\text{-}3)$$

$$W''(x) = \sum_{n=2}^{m} (n)(n-1)A_n\,x^{n-2} \qquad (4\text{-}4)$$

$$W''(x) = \sum_{n=3}^{m} (n)(n-1)(n-2)A_n\,x^{n-3} \qquad (4\text{-}5)$$

$$W''''(x) = \sum_{n=4}^{m} (n)(n-1)(n-2)(n-3)A_n\,x^{n-4} \qquad (4\text{-}6)$$

Using shifting summation indices theory-techniques(Appendix A).

$$\sum_{n=4}^{m} (n)(n-1)(n-2)(n-3)A_n\,x^{n-4} = \sum_{n=0}^{m} (n+4)(n+3)(n+2)(n+1)A_{n+4}\,x^n \qquad (4\text{-}7)$$

$$\text{Assume } P(x) = \sum_{n=0}^{m} P_n\,x^n \qquad (4\text{-}8)$$

So equation (4-1) reduced to

$$\sum_{n=0}^{m} (n+4)(n+3)(n+2)(n+1)A_{n+4}\,x^n + \sum_{n=0}^{m} \alpha\,A_n\,x^n + \beta\Big(\sum_{n=0}^{m} A_n\,x^n\Big)^3$$

$$= \sum_{n=0}^{m} P_n\,x^n \qquad (4\text{-}9)$$

mathematical multiplication of the third term of equation(4-9) gives (*see Appendix C*)

$$\beta \ (\sum_{n=0}^{m} A_n x^n)^3 \ = \sum_{n=0}^{m} AA_n x^n \qquad (4\text{-}10)$$

$$= AA(0) + AA(1)\, x + AA(2)\, x^2 + \ldots$$

Then equation (4-9) becomes

$$\sum_{n=0}^{m} \left((n+4)(n+3)(n+2)(n+1)A_{n+4} + \alpha A_n + \beta AA_n \right) x^n$$

$$= \sum_{n=0}^{m} P_n x^n \qquad (4\text{-}11)$$

equating the both sides will give the RECURRENCE equation.

$$A_{n+4} = \frac{(P_n - \alpha A_n - \beta AA_n)}{(n+4)(n+3)(n+2)(n+1)}$$

or $$A(n+4) = \frac{P(n) - \alpha A(n) - \beta AA(n)}{(n+4)(n+3)(n+2)(n+1)} \qquad (4\text{-}12)$$

This equation gives the relation among A_{n+4} , A_n , α (linear stiffness), β (non linear stiffness) and load $P_n(x)$

4-3 Solution Procedure

4-3-1 Clamped beam / constant load.

In this case the load will take the following form

$P_n(x)$ = constant

$$\sum_{n=0}^{m} P_n x^n = \text{Constant} \tag{4-13}$$

$$P(0) = \text{constant} \tag{4-14}$$

$$\sum_{n=1}^{m} P_n x^n = P(1) = P(2) = \ldots = P(n) = \text{Zero}$$

We will call P(0) = P (4-15)

referring to equation (4-1) with no foundation stiffness ($\alpha=0$ & $\beta=0$)

$$W''''(x) = P \tag{4-16}$$

Now for linear solution

$$W = W_p + W_h \tag{4-17}$$

$$W_h = A_0+A_1x+A_2x^2+A_3x^3 \quad \text{homogenous solution} \tag{4-18}$$

to satisfy equation. (4-16) ;

$$W_p = cx^4 \quad \text{Particular solution} \tag{4-19}$$

$$W'_p = 4\,cx^3$$

$$W''_p = 12\,cx^2$$

$$W'''_p = 24\,cx$$

$$W''''_p = 24c \tag{4-20}$$

from equation (4-16)

$$24\,c = p \Rightarrow c = \frac{P}{24} \tag{4-21}$$

$$\therefore\ W_p = \frac{P}{24}x^4 \tag{4-22}$$

substitute in equation(4-17)

$$W = A_0+A_1x+A_2x^2+A_3x^3 + \frac{P}{24}x^4 \qquad (4\text{-}23)$$

to satisfy Boundary conditions for fixed beam (clamped beam)

$$W(0) = 0 \quad \Rightarrow A_0 = 0$$

$$W'(0) = 0 \quad \Rightarrow A_1 = 0$$

$$W(1) = 0 \quad \Rightarrow A_2 + A_3 + \frac{P}{24} = 0 \qquad (4\text{-}24)$$

$$W'(1) = 0 \quad \Rightarrow 2A_2 + 3A_3 + \frac{P}{6} = 0 \qquad (4\text{-}25)$$

Solving the above simultaneous equations gives :

$$A_2 = \frac{P}{24} \qquad (4\text{-}26)$$

$$A_3 = -\frac{P}{12} \qquad (4\text{-}27)$$

Now These values of A_2 & A_3 are used as the starting values for the nonlinear solution.

By referring to RECURRENCE equation (4-12) with $A_0 = 0$, $A_1 = 0$,A_2 & A_3 from equations(4-26)&(4-27) and P & α & β are given.

we obtain $A_4, A_5 \ldots A_m$.

(n is convergence value of power series degree which will be discussed later) .

So we get the coefficients of power series of equation (4-2) .

Then we satisfy the boundary condition at x = 1 of the clamped beam of non linear equation.

Now we calculate from equation (4-2) $W(1)$ and from equation (4-3) $W'(1)$

if $W(1) \leq$ Tol and

if $W'(1) \leq$ Tol (4-28)

then we obtain the exact solution of non linear equation (4-11)

if Not we restart new values of A_2 & A_3 and so on until we satisfy equation (4-28) .

4-3-2 Clamped beam / cubic load

In This section we choose unsymmetric cubic load applied transversely on a fixed beam this load takes the form

$P(x) = x^3 - 2x^2 + x$ (4-29)

See figure (3-10)

So $P(0) = 0.0$

$P(1) = 1.0$

$P(2) = -2.0$

$P(3) = 1.0$

$P(4) = P(5) = \ldots\ldots = P(n) = 0$ (2-30)

Following the same procedure as the previous section

$W = W_h + W_p$

with particular solution as

$W_p = c_7x^7 + c_6x^6 + c_5x^5 + c_4x^4 + c_3x^3 + c_2x^2 + c_1x + c_0$ (4-31) solving as equation (4-20) & (4-21) with

$$W''''_p = P_n(x) = x^3 - 2x^2 + x \tag{4-32}$$

$$W''''_p = 840\, c_7 x^3 + 360\, c_6 x^2 + 120\, c_5 x + 0$$

$$= x^3 - 2x^2 + x$$

we get $c_7 = \frac{1}{840}$ $\qquad c_5 = \frac{1}{120}$

$$c_6 = \frac{-1}{180} \qquad c_4 = c_3 = c_2 = c_1 = c_0 \tag{2-33}$$

so

$$W_p = \frac{1}{840}x^7 - \frac{1}{180}x^6 + \frac{1}{120}x^5 \tag{4-34}$$

then the general solution

$$W = (A_0 + A_1 x + A_2 x^2 + A_3 x^3) + (\frac{1}{840}x^7 - \frac{1}{180}x^6 + \frac{1}{120}x^5) \tag{4-35}$$

Now boundary conditions at x=0

$W(0) = 0 \quad \Rightarrow \quad A_0 = 0$

$W'(0) = 0 \quad \Rightarrow \quad A_1 = 0$

so

$$W = (A_2 x^2 + A_3 x^3) + (\frac{1}{120}x^5 - \frac{1}{180}x^6 + \frac{1}{840}x^7) \tag{4-36}$$

Boundary conditions at (x = 1.0)

$W(1) = 0.0$

$W'(1) = 0.0$

we get $A_2 = 0.004762$

$$A_3 = 0.00870 \tag{4-37}$$

Starting with these values of A_2 & A_3 to solve non-linear equation (4-11) by using recurrence equation (4-12) with given α & β to get A_4, A_5... A_n, under the conditions of satisfy boundary conditions of non linear equation

$$W(1) \leq \text{Tol} \quad \text{and}$$

$$W'(1) \leq \text{Tol} \qquad (4\text{-}38)$$

4-3-3 Clamped beam / sinusoidal load

In this case a symmetric load is taken in the form

$$P(x) = \sin \pi x \qquad (4\text{-}39)$$

writing sin πx in Taylor series *(see Appendix B)*

$$\sin \pi x = \pi x - \frac{(\pi x)^3}{3!} + \frac{(\pi x)^5}{5!} - \frac{(\pi x)^7}{7!} + \frac{(\pi x)^9}{9!} + \qquad (4\text{-}40)$$

following the same procedure of previous sections to get

$$W_p = c_1 \sin \pi x + c_1 \cos \pi x \qquad (4\text{-}41)$$

$$W''''_p = \sin \pi x \qquad (4\text{-}42)$$

$$\text{we get} \quad c_1 = \frac{1}{\pi^4} \qquad c_2 = 0.0 \qquad (4\text{-}43)$$

$$W = W_h + W_p$$

$$= (A_0 + A_1 x + A_2 x^2 + A_3 x^3) + \left(\frac{1}{\pi^4} \sin \pi x\right) \qquad (4\text{-}44)$$

Boundary conditions at $x = 0$

$$W(0) = 0 \quad \Rightarrow \quad A_0 = 0.0$$

$$W'(0) = 0 \quad \Rightarrow \quad A_1 = 0.0 \qquad (4\text{-}45)$$

$$W = A_2 x^2 + A_3 x^3 + \frac{1}{\pi^4} \sin \pi x \qquad (4\text{-}46)$$

Boundary condition at x= 1.0

$W(1) = 0.0 \quad \Rightarrow \quad A_2 + A_3 = 0$

$W'(1) = 0.0 \quad \Rightarrow \quad 2A_2 + 3A_3 - \frac{1}{\pi^4} = 0$

we get $A_2 = -\frac{1}{\pi^3} \quad A_3 = \frac{1}{\pi^3}$ (4-47)

starting with those values of A_2 & A_3 to get $A_4, A_5, \ldots, A_n$ from recurrence equation (4-12) with known values of α & β, satisfying boundary conditions of non-linear equation(4-11) to achieve the exact solution within given tolerance for

$W(1) \leq$ Tol and

$W'(1) \leq$ Tol (4-48)

if not, restart with new values of A_2 & A_3 until achieve equation (4-48).

4-3-4 Simply Supported Beam / constant load.

The same procedure is carried out as in section 4-3-1, the only difference between them is the boundary conditions

$W(x) = W_h + W_p$

$$= A_0 + A_1x + A_2x^2 + A_3x^3 + \frac{P}{24}x^4 \quad (4\text{-}49)$$

$W(0) = 0 \quad \Rightarrow \quad A_0 = 0$

$W''(0) = 0 \quad \Rightarrow \quad A_2 = 0$ (4-50)

$$W(x) = A_1x + A_3x^3 + \frac{P}{24}x^4 \quad (4\text{-}51)$$

$W(1) = 0 \Rightarrow A_1 + A_3 + \frac{P}{24} = 0.0$

$W''(1) = 0 \Rightarrow 6A_3 + \frac{P}{2} = 0$ (the moment at both ends are zero).

$$A_3 = -\frac{P}{12}$$

$$A_1 = \frac{P}{24} \tag{4-52}$$

by using Recurrence equation (4-12) we get

$A_4, A_5, \ldots, A_n$

So A_2 & A_3 in equation (4-52) are used as starting values for solving nonlinear equation (4-11) , under the condition to satisfy boundary conditions at x = 1.0

$W(1) \leq \text{Tol}$ and

$$W''(1) \leq \text{Tol} \tag{4-53}$$

we obtain the solution.

4-3-5 Simply Supported beam / cubic load

As in section (4-3-4) equation (4-35)

$$W = (A_0 + A_1x + A_2x^2 + A_3x^3) + (\frac{1}{840}x^7 - \frac{1}{180}x^6 + \frac{1}{120}x^5)$$

Boundary conditions at x = 0.

$W(0) = 0 \Rightarrow A_0 = 0$

$$W''(0) = 0 \Rightarrow A_2 = 0 \tag{4-54}$$

$$W = (A_1x + A_3x^3) + (\frac{1}{840}x^7 - \frac{1}{180}x^6 + \frac{1}{120}x^5) \tag{4-55}$$

Boundary conditions at x = 1.0

$W(1) = 0.0$ gives $A_1 + A_3 + \frac{1}{840} - \frac{1}{180} + \frac{1}{120} = 0$

$W(1) = 0.0$ gives $6A_3 + \frac{42}{840} - \frac{30}{180} + \frac{20}{120} = 0$

$$\Rightarrow A_3 = -\frac{1}{120} = -0.0083333$$

$$A_1 = +0.004365 \qquad (4\text{-}56)$$

from equation (4-12) (Recurrence equation) to get $A_4, A_5, \ldots, A_n$ that satisfy the boundary conditions of non linear equation (4-11)

$W(1) \leq \text{Tol}$ and

$W''(1) \leq \text{Tol}$ (4-57)

to get exact solution.

4-3-6 Simply Supported Beam / sinusoidal load.

from equation (4-44) we have

$$W = (A_0 + A_1 x + A_2 x^2 + A_3 x^3) + (\frac{1}{\pi^4} \sin \pi x)$$

boundary condition at x = 0.0

$W(0) = 0.0 \Rightarrow A_0 = 0$

$W''(0) = 0.0 \Rightarrow A_2 = 0$ (4-58)

So $W = A_1 x + A_3 x^3 + \frac{1}{\pi^4} \sin \pi x$ (4-59)

Boundary condition at x = 1.0

$W(1) = 0 \Rightarrow A_1 + A_3 = 0$

$W''(1) = 0 \Rightarrow 6A_3 - \frac{1}{\pi^2} \sin \pi(1) = 0$

$$\Rightarrow A_3 = 0$$

$$A_1 = 0 \qquad (4\text{-}60)$$

Also these starting values of recurrence equation (4-12) to get power series coefficient $A_4, A_5, \ldots, A_n$, with satisfying boundary conditions of non linear equation (4-11) at $x = 1.0$

$W(1) \leq \text{Tol}$ and

$$W''(1) \leq \text{Tol} \qquad (4\text{-}61)$$

to achieve exact solution .

Beam Type	Load	symmetry	Initial Values			
			a_0	a_1	a_2	a_3
Clamped	Constant (P)	symmetric	0	0	p/24	-p/12
Clamped	Cubic X^3+2X^2+X	unsymmetric	0	0	0.004762	0.008700
Clamped	$\sin \pi x$	symmetric	0	0	$-1/\pi^3$	$+1/\pi^3$
Simply Supported	Constant (P)	symmetric	0	p/24	0	-p/12-
Simply Supported	Cubic X^3+2X^2+X	unsymmetric	0	0.004365	0	-0.008333
Simply Supported	$\sin \pi x$	symmetric	0	0	0	0

Table (4-1) Summary of initial values of a_0, a_1, a_2 & a_3 for different types of beams and loads

4-4 Convergence of power series.

To determine the degree of power series that converges to the solution without any significant difference as the degree n increases we choose the midspan of the beam at x=0.5 as the study point, then we fixed the values of α & β linear and non linear stiffness of foundation at constant load / clamped beam, then we solved for deflections and stresses with different values of n starting from n=8 to n=60, we plotted the deflections vs n (power series degree) and stresses vs n, from the curves we calculated the convergence value of n see Fig (5-1a) & (5-1b).

n	Deflection at x=0.5	Stress at x=0.5	n	Deflection at x=0.5	Stress at x=0.5
8	0.04629216	-0.3263889	26	0.04480822	-0.5875433
10	0.03246656	-0.3036057	28	0.04480822	-0.5875433
12	0.02689526	-0.3174513	30	0.04480822	-0.5875433
14	0.04443071	-0.5017268	32	0.04480822	-0.5875433
16	0.04480888	-0.5868502	34	0.04480822	-0.5875433
18	0.04480824	-0.5875162	36	0.04480822	-0.5875433
20	0.04480822	-0.5875429	38	0.04480822	-0.5875433
22	0.04480822	-0.5875433	40	0.04480822	-0.5875433
24	0.04480822	-0.5875433			

Table (4-2) Convergence of power series degree n for deflection and stress of clamped beam / constant load at mid point (x=0.5)

4-5 Harmonic Balance method

This method is used to solve the nonlinear differential equation (4-1) for special case simply supported beam under sinusoidal load $P(\zeta) = \sin \pi \zeta$.

Recall equation (4-1)

$W''''(x) + \alpha W(x) + \beta W^3(x) = P(x).$

let $P = \sin \pi \zeta$ (4-62)

Assume $W = A \sin \pi \zeta$ (4-63)

substitute in equation (4-1) we get

$$\pi^4 A \sin \pi \zeta + \alpha A \sin \pi \zeta + \beta A^3 \sin^3 \pi \zeta = \sin \pi \zeta \tag{4-64}$$

from Trigonometric functions we get

$$\sin^3 \pi \zeta = \frac{1}{4}(3\sin \pi \zeta - \sin 3\pi \zeta) \tag{4-65}$$

substitute in equation (4-64) with $\alpha = 1.0$ & $\beta = 1.0$

$$^4A \sin \pi \zeta + A \sin \pi \zeta + A^3 \left(\frac{1}{4}(3\sin \pi \zeta - \sin 3\pi \zeta)\right) = \sin \pi \zeta \tag{4-66}$$

by equating coefficients of $\sin \pi\zeta$ of both sides we obtain

$$\pi^4 A + A + A^3 \left(\frac{3}{4}\right) = 1.0$$

$$\frac{3}{4} A^3 + (\pi^4 + 1)A = 1.0$$

$$0.75 A^3 + 98.41 A = 1.0 \tag{4-67}$$

solving for A we get

$A = 0.01016$

$$\text{So } W(\zeta) = 0.01016 \sin \pi \zeta \tag{4-68}$$

for $\alpha = 10.0$ & $\beta = 10.0$ we get.

$$W(\zeta) = 0.00931 \sin \pi \zeta \tag{4-69}$$

plotting these function equations (4-68 & 4-69) for (ζ 0 to 1) we obtain Fig.(5-28 a&b)

4-6 Deflection Calculations

As we get the power series coefficient for convergence n, we calculate the deflection from equation (4-12)

$W(x) = \sum_{n=0}^{m} A_n x^n$ (where m is the converged power series exponent).

we divided the beam into 10 elements from 0.0,0.1,0.2 ,.... to 1.0 at each point we get the deflection value then it plotted Versus beam axis .

4-7 Stress Calculations

As we know from elasticity & strength of material is that

$$\sigma_x = \frac{MC}{I} \qquad (4\text{-}70)$$

where σ_x is the maximum normal stress in x - direction

M: is bending moment.

I : area moment of inertia .

C: the distance from center to the top of cross section of the beam

Also we know that

$$M = EI\frac{d^2W}{dx^2} \qquad (2\text{-}71)$$

where E is the modulus of elasticity of beam material.

So $\sigma_x = \dfrac{EIW''C}{I} = EC\ W''(x)$

$$\frac{\sigma_x}{EC} = W''(x) \tag{4-72}$$

since EC is constant.

So if we plot W'' (x) from equation (4-4) Versus beam axis we get the normal stress divided by EC. Subroutines are set to calculate the deflection W (x), slope, W' (x) and second derivative W'' (x) in FORTRAN .

Also Factorial of integer & non linear term $(W(x))^3$ subroutines are programmed.

Small step size was used to search for suitable values of A_2 & A_3 with given range for clamped beam while the searching for (A_1 & A_3) for simply supported beam.

The region is initially taken near the linear solution.

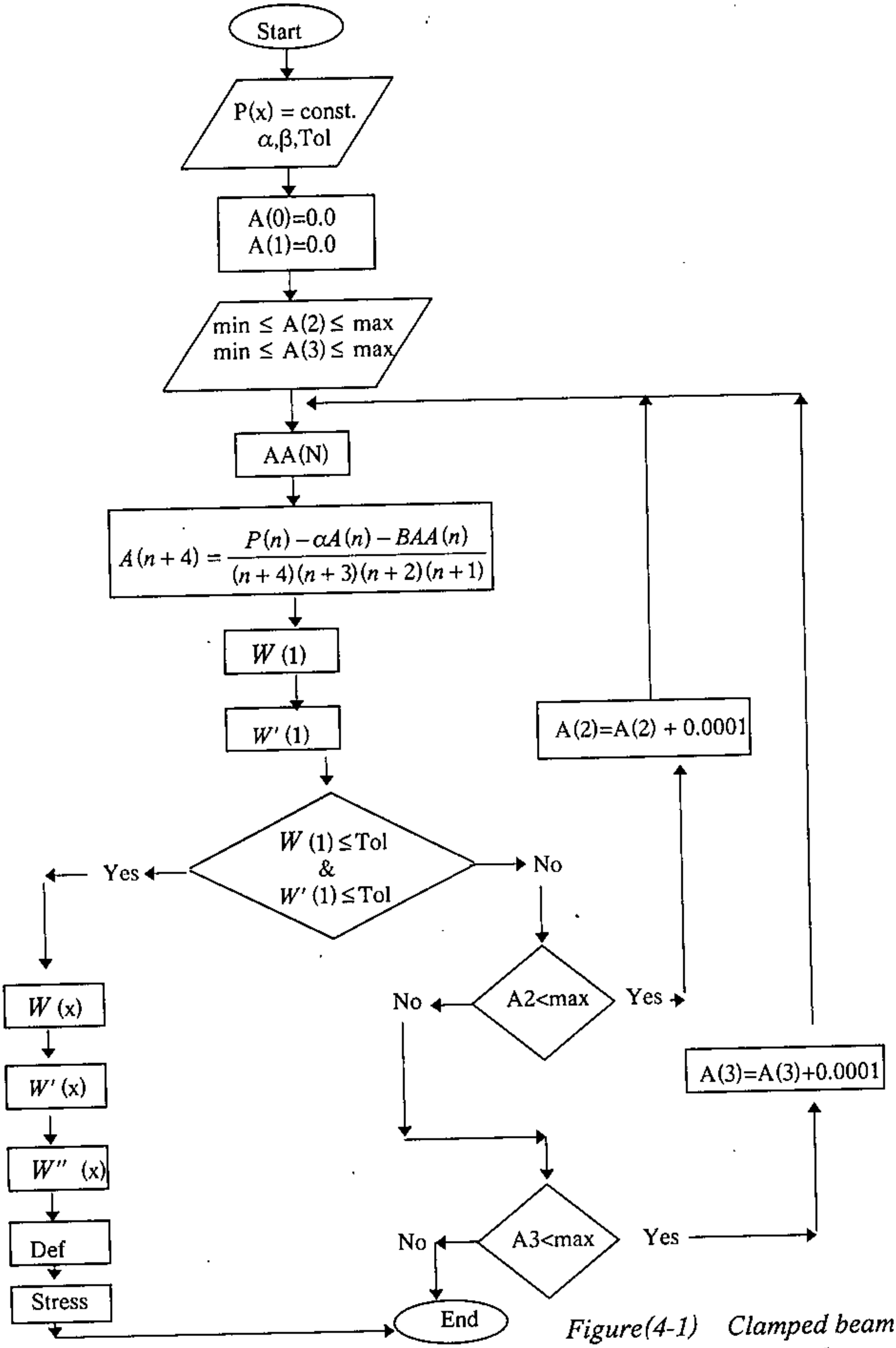
Constant load are taken as 1,10,50.....

The value of linear stiffness α: 0,1,10,20

The non linear stiffness β: 0,1,10,100,1000 , ...

Computer programs can be run after specifying the values of all parameters to obtain the exact power series coefficients and then deflections & stresses at each point .

4-8 FLOW CHART



Figure(4-1) Clamped beam / const. load flow chart

CHAPTER FIVE

RESULTS AND DISCUSSION

5-1 Introduction

The main objective of the present study is to solve the large deflections of clamped & simply supported beams resting on non-linear elastic foundation by power series method which is very effective in linear problems.

Results obtained from this study proved that the power series method can be used efficiently for nonlinear problems as well as for linear problems. Computer programs in FORTRAN language (Appendix D) are set to solve the fourth order non linear differential equation which governed the large deflections of beams subjected to different kinds of loads . Double precision numbers of all data are taken in consideration to get high accuracy Sample of data is attached in Appendix E .

All parameters that affect the deflections and stresses of the beam as linear stiffness (α), non-linear stiffness (β),load shape, power series degree and boundary conditions are studied in this chapter .

Comparison test was made with the Harmonic Balance method for nonlinear large deflections revealed the effectiveness of power series method .

5-2 Results :

Solutions of the fourth order non linear differential equation at certain value of α & β at given load for clamped & simply supported beams are presented in figures at the end of this chapter.

Convergence of power series degree (n) was the first step of the study to determine the suitable degree at which no more significant effect on the solution of the problem occurs. Deflections and bending stresses at midspan of clamped beam (x=0.5) are plotted at different values of n , starting from 8 to 40 , Figures (5-1 a & b) show the results at which n=16 or more yields no significant effect on deflections while at n=18 and up no change on bending stress values is found, the study was taken at constant load p=10 , α=1.0 and β=1.0 .

Clamped beam under constant load of different values starting from 1,10,20,... up to 10000 at fixed value of $\alpha = 0.0$ & $\beta = 0.0$ which represents the linear solution since no stiffness, is plotted in Figures (5-2 a & b) that show the effect of loads on deflections and stresses .

Furthermore , another values of α & β were studied as shown in Figures (5-3 a & b) for $\alpha = 0, \beta = 10$, while Figure (5-4 a& b) shows the deflections and bending stress for $\alpha = 10, \beta = 0$, and Figure (5-4 a & b) was plotted for α=10 & $\beta = 0.0$.

Investigating the effect of α which takes different values from 1 to 100 at constant load p =10 at $\beta = 0$ and $\beta = 10$ is represented respectively in Figure (5-6 a & b) and Figure (5-7 a & b).

Changing the values of β from 1 to 1000 at constant load p =10 and different value of $\alpha = 1$ & $\alpha = 10$ are plotted for deflections and stresses in Figures (5-8 a & b) and (5-9 a & b) respectively.

Symmetrical shapes for deflections and stresses are obtained in all the above mentioned figures since the load is symmetric, at the same time boundary conditions are satisfied at both ends of the beam.

Another load shape having investigated in this study is the cubic unsymmetric load in the form of $x^3 - 2x^2 + x$ which is represented in Figure (5-10).

Deflections and bending stresses of clamped beam due to cubic load are plotted in Figure (5-11 a & b) for $\beta = 0$ with different values of α (from 1 to 15) , while Figure (5-12 a & b) the case when $\beta = 10$.

Changing the values of β from 1 to 10^6 at constant values of linear stiffness $\alpha = 0$ and $\alpha = 10$ are represented in Figures(5-13 a & b) and (5-14 a & b) respectively, as we see from figures no significant changes in deflections and stresses are encountered.

Sinusoidal load in the form of $\sin \pi x$ which is half sine wave is another type of load that studied in this work, load shape is plotted in Figure (5-15), when this load is exerted on clamped beam at different values of α from 1 to 200 at constant values of $\beta=0$ and $\beta=10$ non linear deflections and stresses are clearly observed in Figure (5-16 a & b) and (5-17 a & b) respectively.

Investigating the effect of changing the non linear stiffness β at constant value of $\alpha= 0$ & $\alpha=10$ is represented in Figures(5-18 a & b) and (5-19 a & b) respectively small changes in deflections and stresses are noticed.

Simply supported beams subjected to different types of loads are analyzed in this work, deflections and stresses due to constant load changing at $\alpha=0$ & $\beta=0$ is plotted in Figure (5-20 a & b), while Figure (5-21 a & b) shows the results when $\alpha= 10$ and $\beta=10$. The difference between the two figures are observed clearly.

The effect of changing α at constant load p=10 and $\beta=10$ is plotted in Figure (5-22 a & b), symmetrical shapes are seen in figures.

Figure (5-23 a & b) shows the deflections and stresses due to changing β at constant load p=10 and $\alpha=10$.

Simply supported beams under cubic load is analyzed in Figures (5-24 a & b) and (5-25 a & b) for α variable & $\beta = 10$,and β variable & $\alpha =1$ respectively, unsymmetrical shapes are noticed in figures with small changes in deflections and stresses.

The last case which was investigated was simply supported beam subjected to sinusoidal load, changing α from 1 to 100 at constant value of $\beta = 10$ is plotted in Figure(5-26 a & b), while the effect of changing β from 1 to 10^5 at constant value of α =10 on deflections and stresses is represented in Figure (5-27 a & b).

For all cases of simply supported beams, the boundary conditions of deflections and moment at both ends are satisfied for non linear problem.

Comparison test was made for simply supported beam / sinusoidal load at α = 1.0 , 10.0 and β = 1.0,10.0 between power series method and harmonic balanced method, results obtained from the two method are plotted in Figure (5-28 a & b), the results found to be in very good agreements.

5-3 Discussion

5-3-1 Effect of power series degree(n)

As we know the power series has an infinite power degree n, but any study makes searching on this number to specify where this number has no significant influence on the solution.

In my study, I did the convergence plot in Figure (5-1 a & b) which illustrated how much the deflections and stresses are changed at midspan of beam(x= 0.5) with respect to the power series degree n,. from Figure (5-1 a) we found the deflection has no significant change at n = 16 and more, while Figure (5-1 b) shows at n=18 no more significant change in stress at x=0.5 , so n=20 is taken the best value of n (power series degree) for all cases in my study.

5-3-2 Choice of beam / Boundary conditions

5-3-2-1 Clamped beams

This kind of beams has two boundary conditions at each end where the deflection and the slope are zero.

We found the deflections and stresses are symmetric for symmetric loads such as constant and sinusoidal loads, the maximum values are at midspan of beam (x=0.5) which is a normal expected condition .

For non symmetric loads as cubic load the situation is different and this can be observed from Figures (5-11) to (5-14).

5-3-2-2 Simply supported beams

The deflection and moment are zero at each end of this kind of beams also the maximum values of deflections and stresses are at x=0.5 for symmetric loads as in sinusoidal and constant loads while it is not necessarily the case for unsymmetric load, this point is illustrated in Figure (5-25).

5-3-3 Effect Of Loads

Three different loads are chosen in this study, a constant load where P(0) = P as seen from figures where the load increases, the deflections and stresses increases which is expected.

The unsymmetric cubic load in the form of $P(x) = x^3 - 2x^2 + x$ see Figure (5-10) has unsymmetrical characteristics in deflections and stresses.

The third load is sinusoidal load which is $P(x) = \sin \pi x$ that transformed in Taylor series equation (4-40) has symmetry characteristics in deflection and stress .

Deflections and stresses are plotted for all loads at α=0.0 & β=0.0 which is linear solution and also plotted for non linear solution with $\alpha \neq 0.0$ & $\beta \neq 0.0$,the differences are very obviously noticed.

5-3-4 Effect of linear stiffness (α)

The increase of the linear stiffness of foundation α increases the required load for beam deflection when other parameters are constants, this happens actually because the strain energy of foundation increases and then the structure becomes more hardened. So a more load is required to deflect the beam, this appears clearly in Figures (5-6),(5-7),(5-11),(5-12) and (5-13) for clamped beams, and in Figures (5-22),(5-24) and (5-26) for simply supported beams, i.e. the deflections decrease as α increases.

5-3-5 Effect of non linear stiffness (β)

As the non linear stiffness β increases the deflections and stresses decrease when other parameters remain constants for each type of load, but this reduction is less than for corresponding decreasing effect of α for the same values of loads , which means that the effect of non linear term β is less than the effect of α since β is the coefficient of W^3, while α is the coefficients of W in the governing equation . This is illustrated in Figures (5-9),(5-13),(5-19) and (5-27).

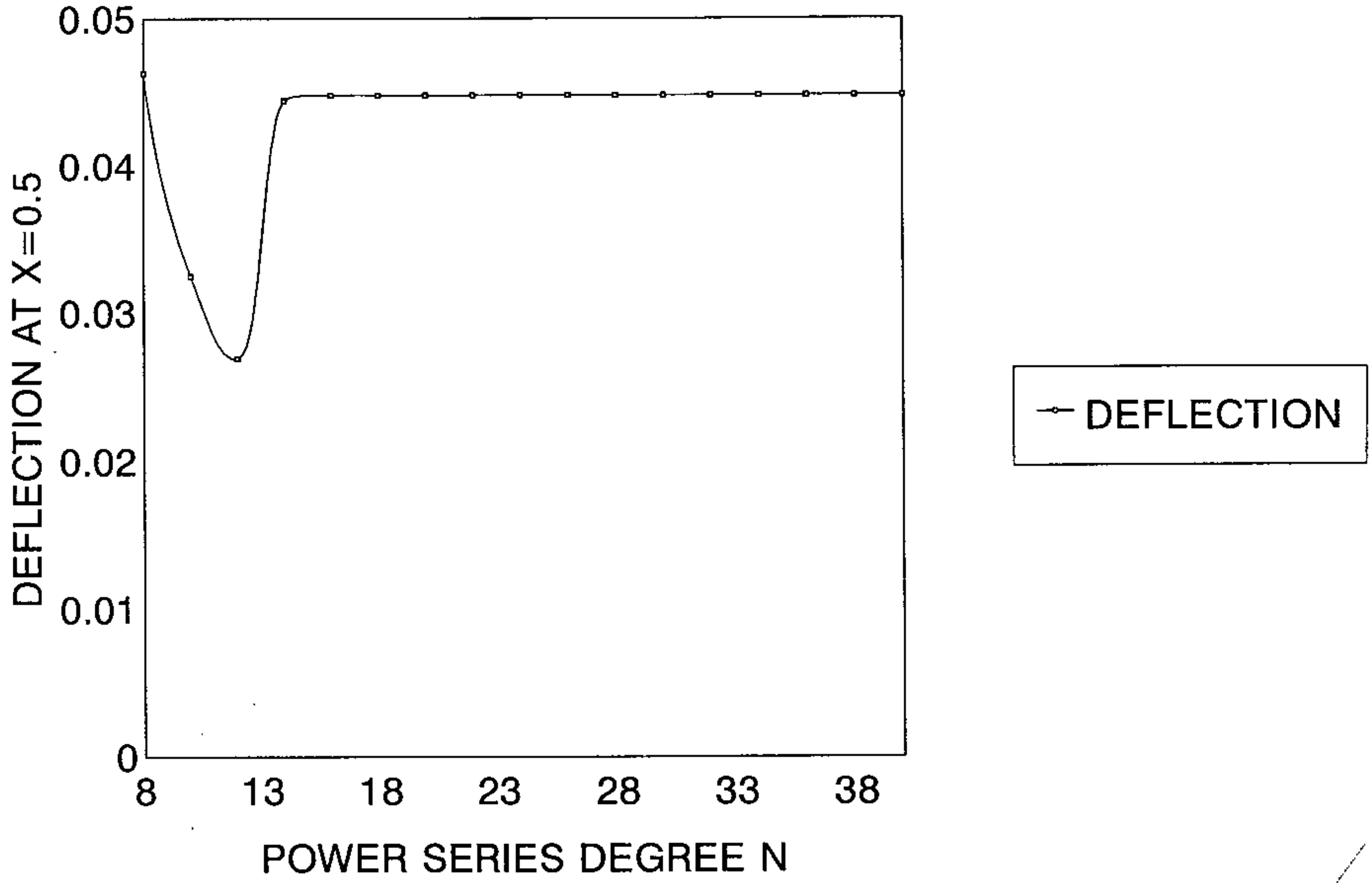


Fig.(5-1a) Convergence of power series degree,Deflection at (X=0.5) versus degree n with load=10, alfa=1, beta=1

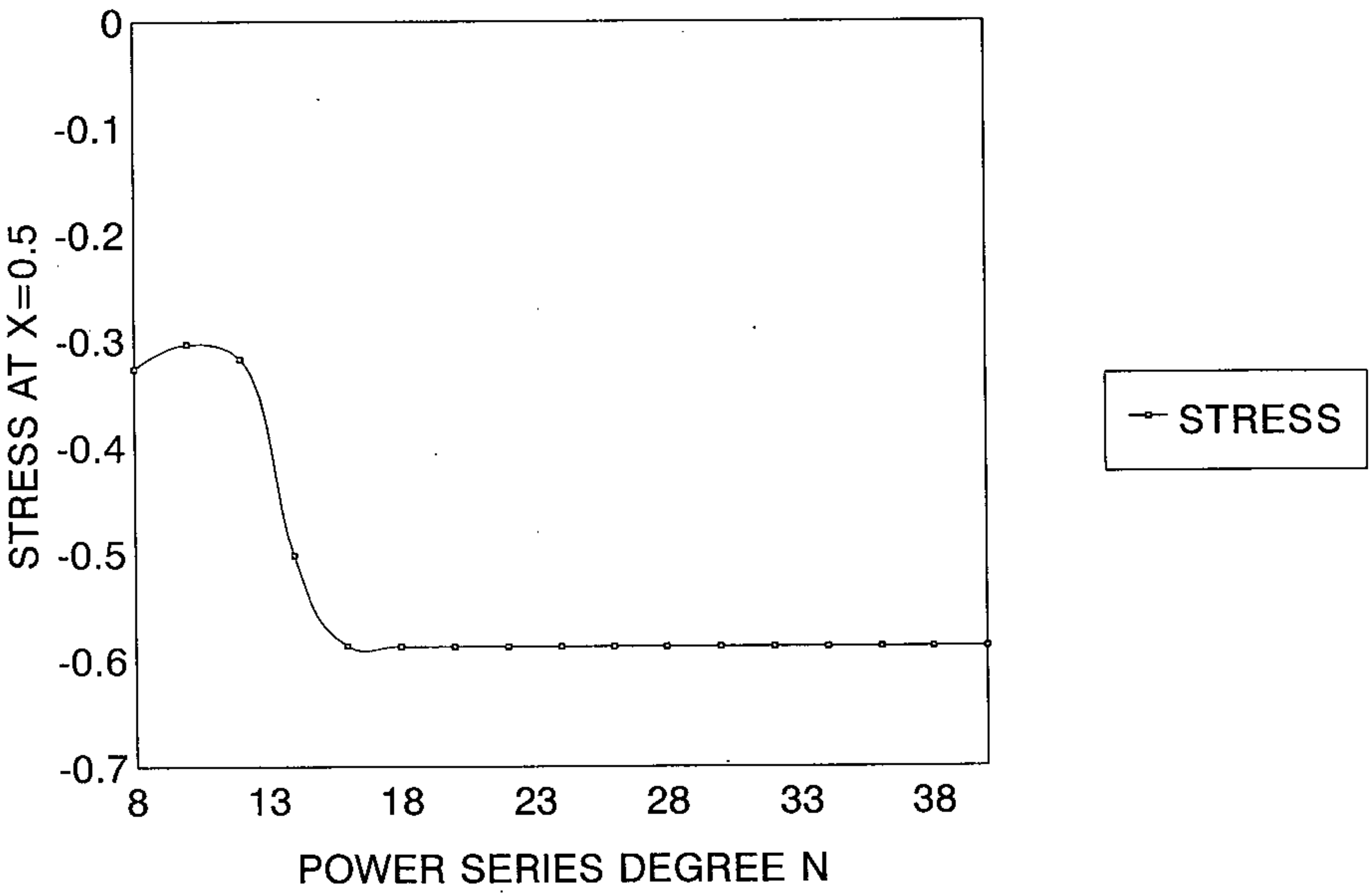


Fig.(5-1b) Convergence of power series degree, Stress at (X=0.5) versus degree n with load=10, alfa=1, beta=1

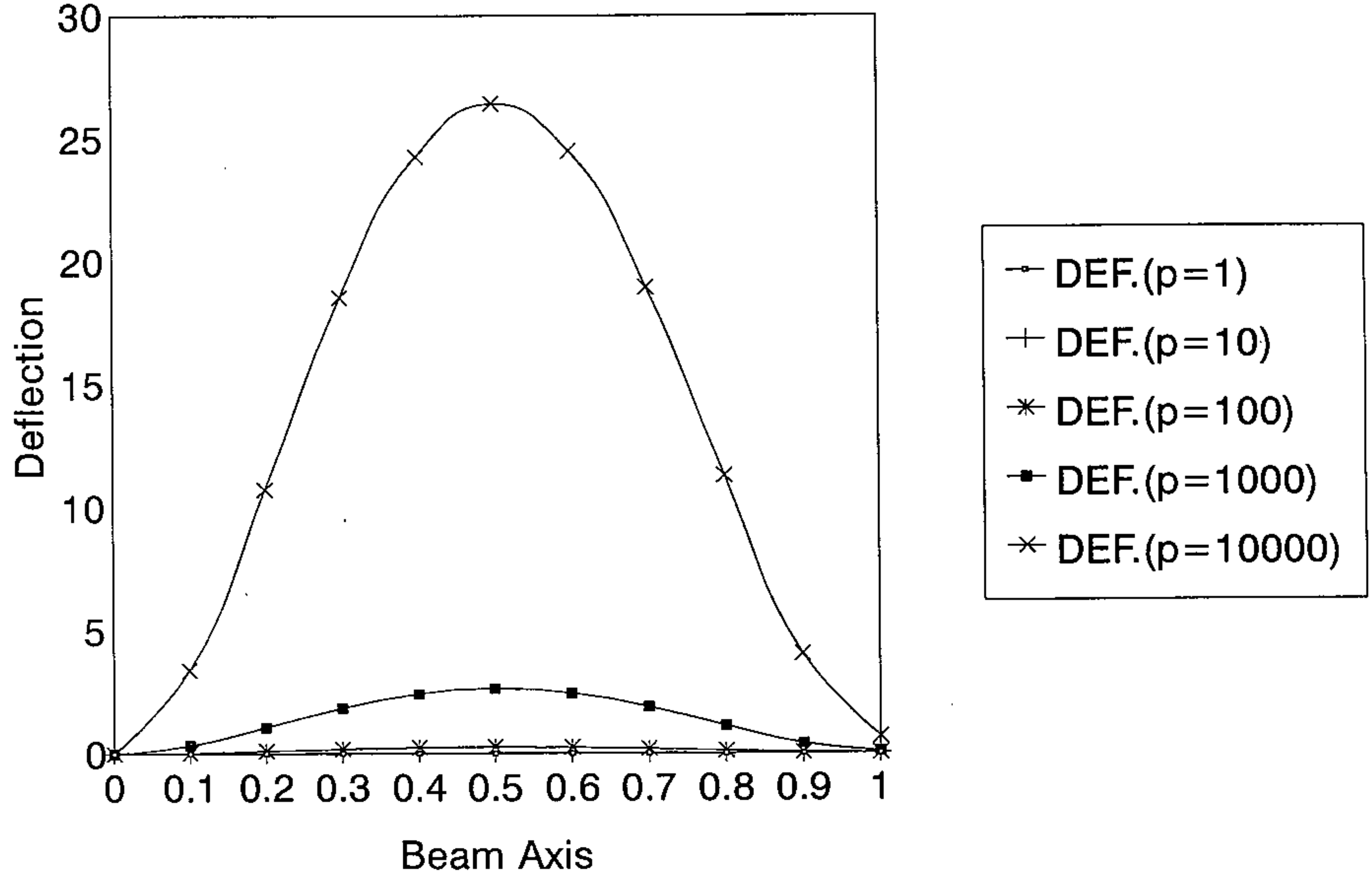


Fig.(5-2a)Deflection versus beam axis for clamped beam/ constant load,with laod variable,alfa=0,beta=0

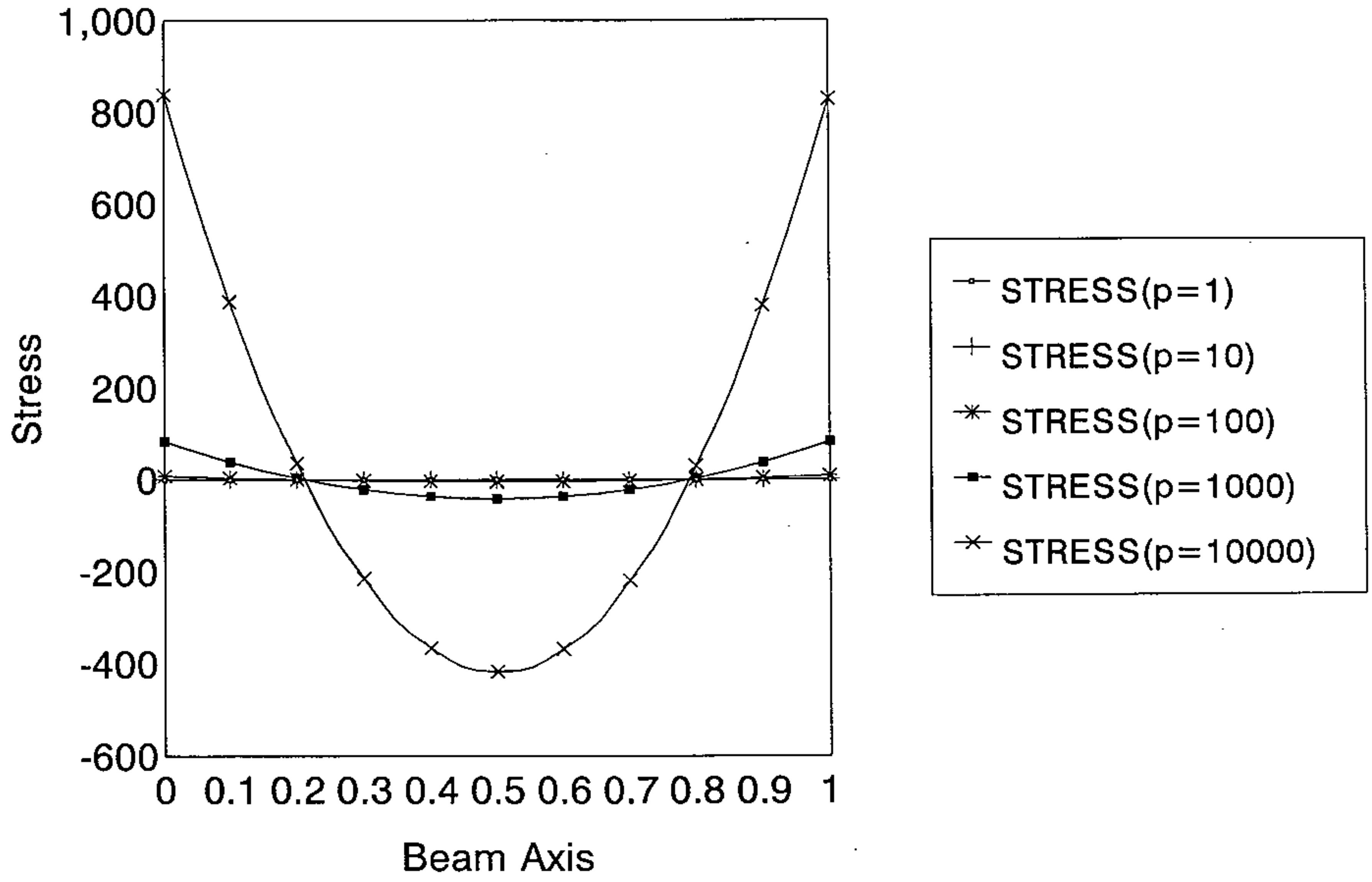


Fig.(5-2b) Stress versus beam axis for clamped beam/ constant load with load variable,alfa=0 ,beta=0

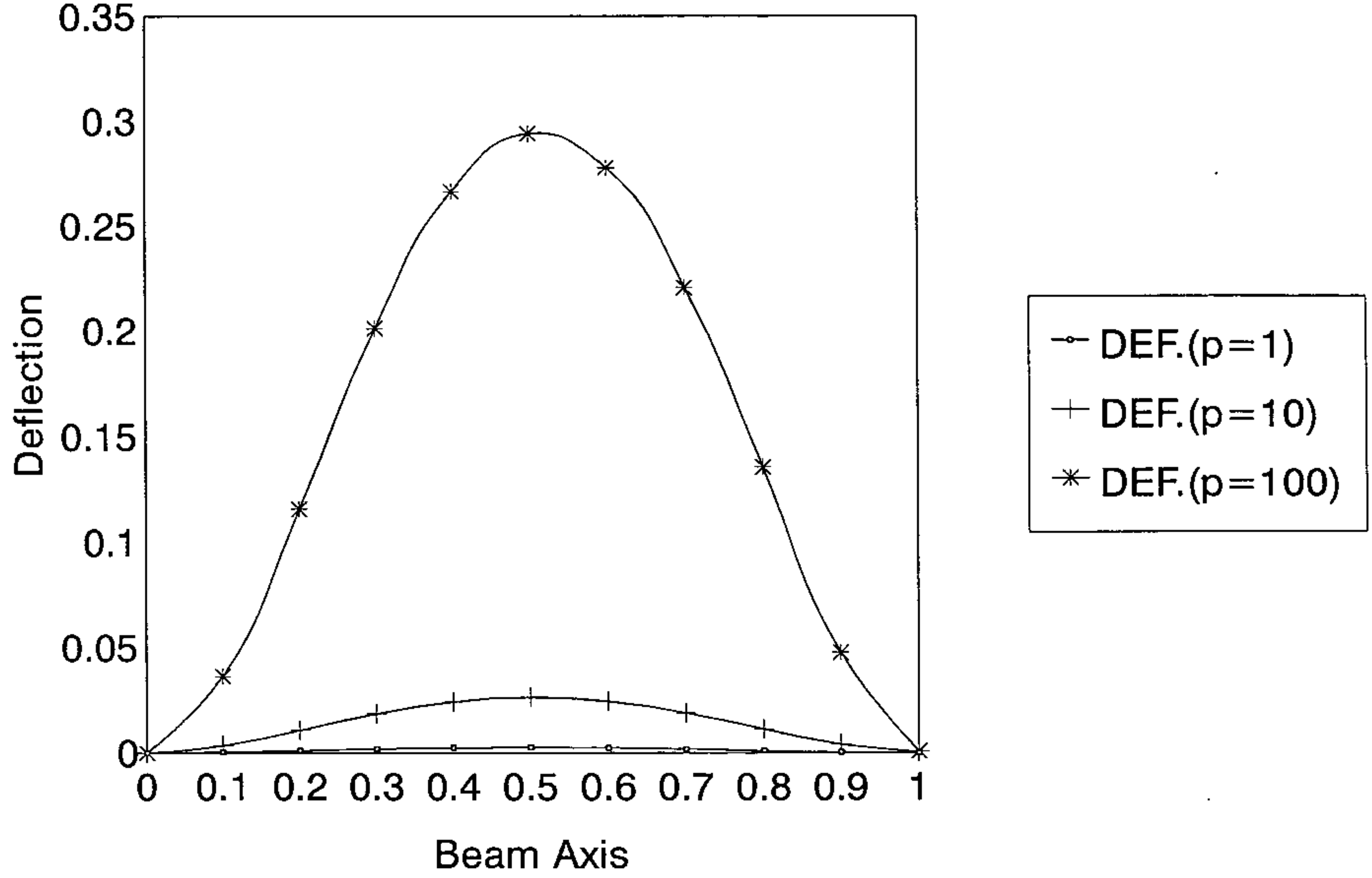


Fig.(5-3a) Deflection versus beam axis for clamped beam/ constant load with load variable,alfa=0,beta=10

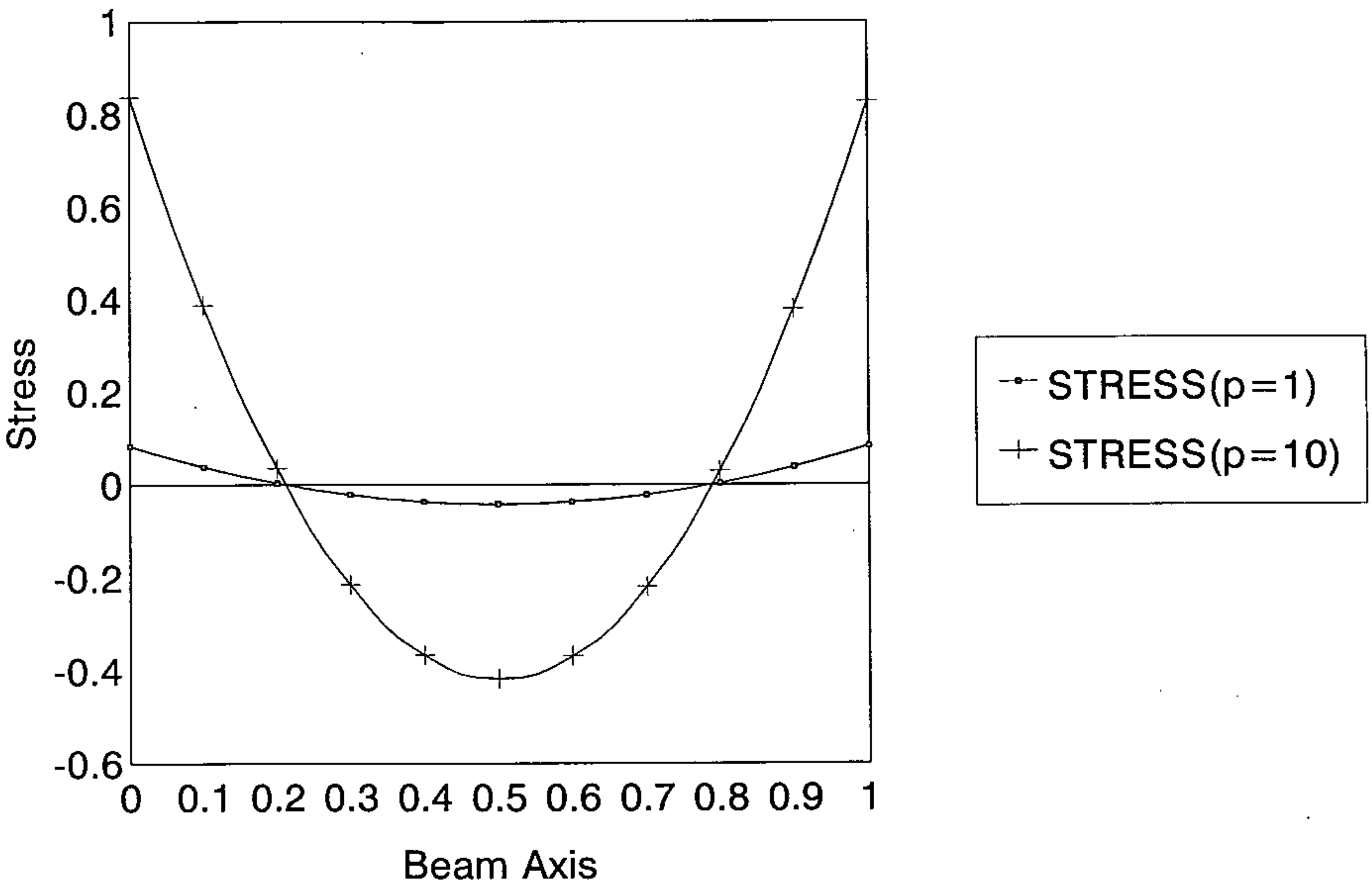


Fig.(5-3b) Stress versus beam axis for clamped beam/ constant load with load variable,alfa=0,beta=10

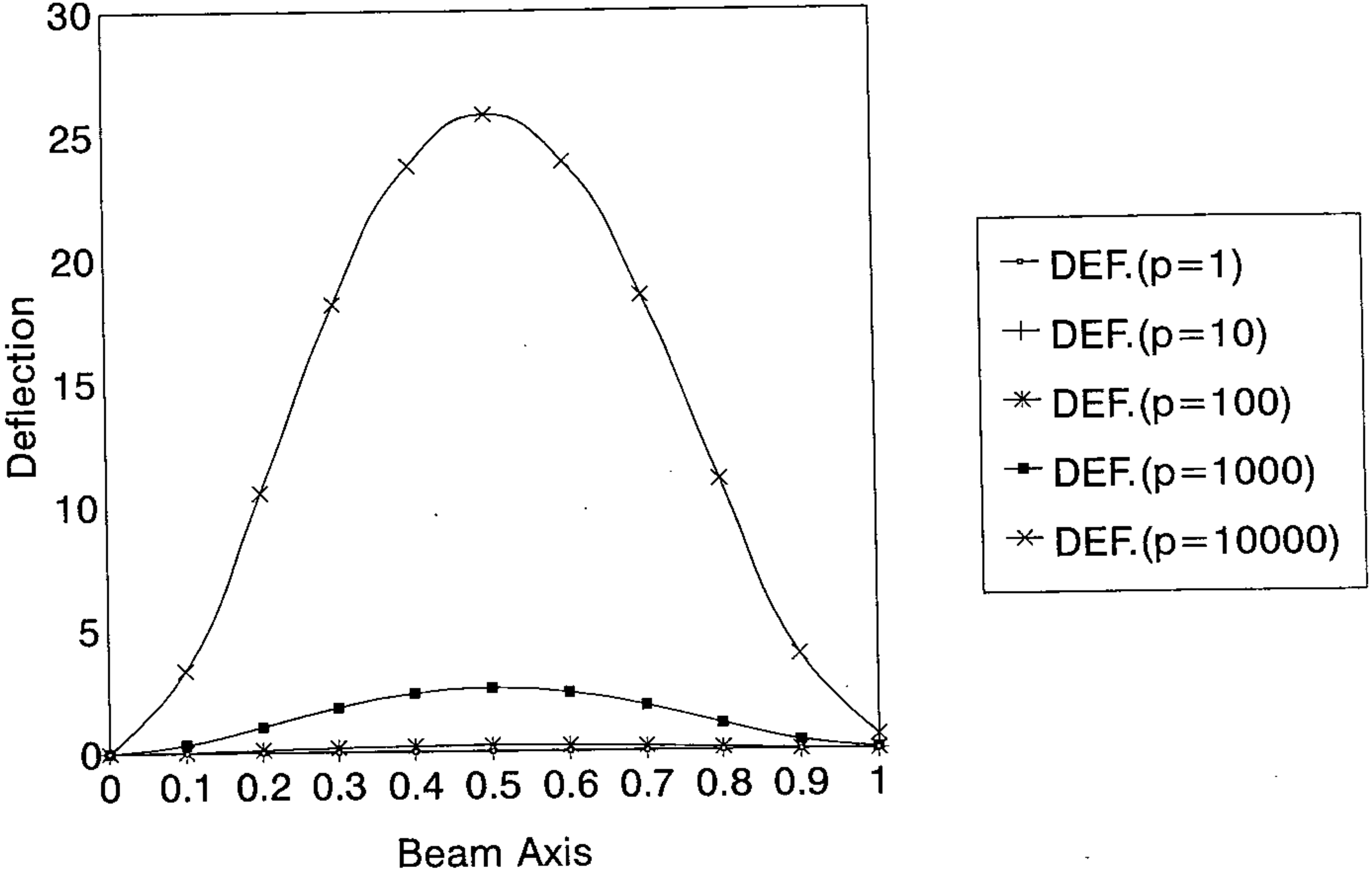


Fig.(5-4a) Deflection versus beam axis for clamped beam/ constant load with load variable,alfa=10,beta=0

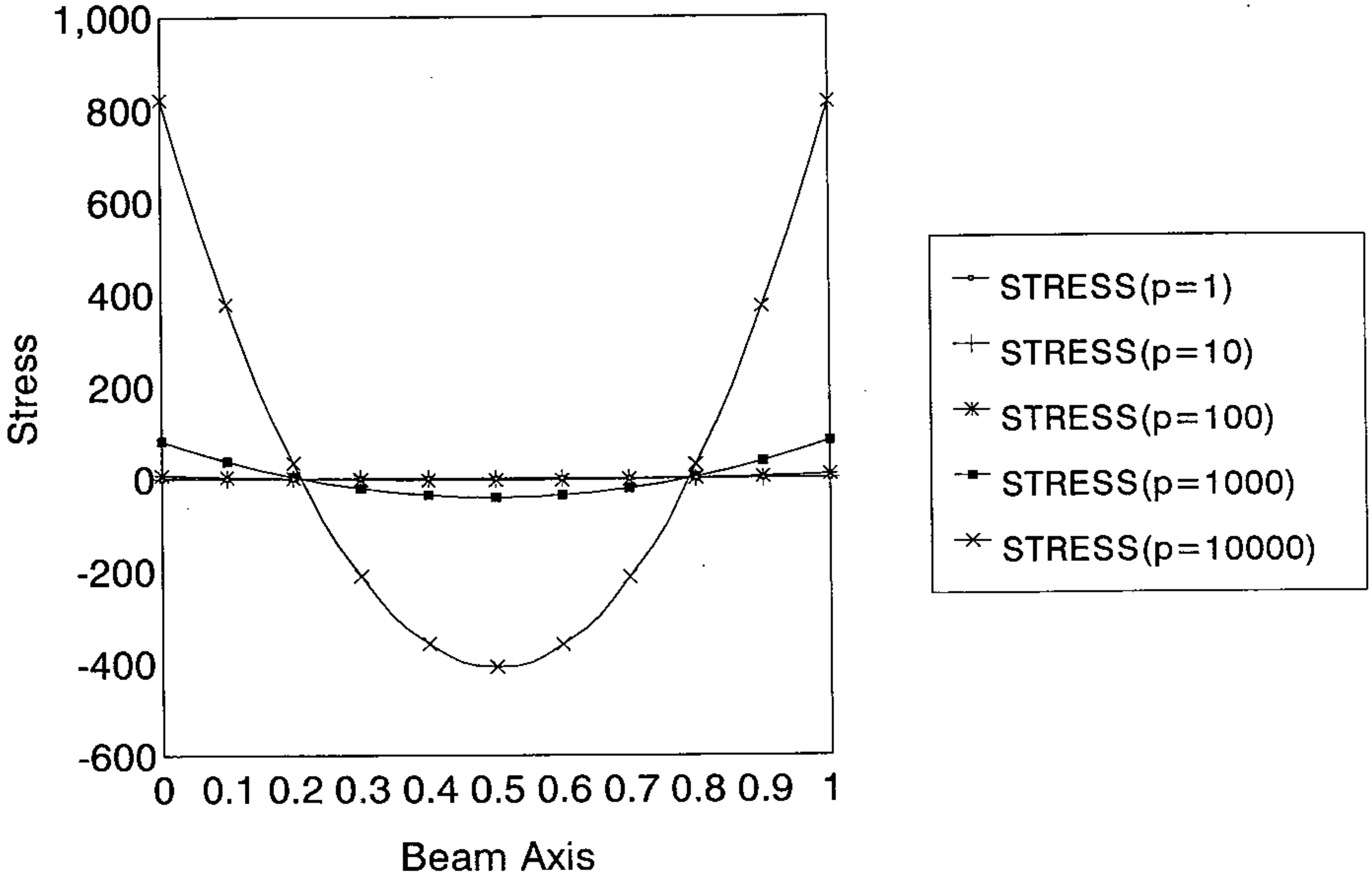


Fig.(5-4b) Stress versus beam axis for clamped beam/ constant load with load variable,alfa=10 ,beta=0

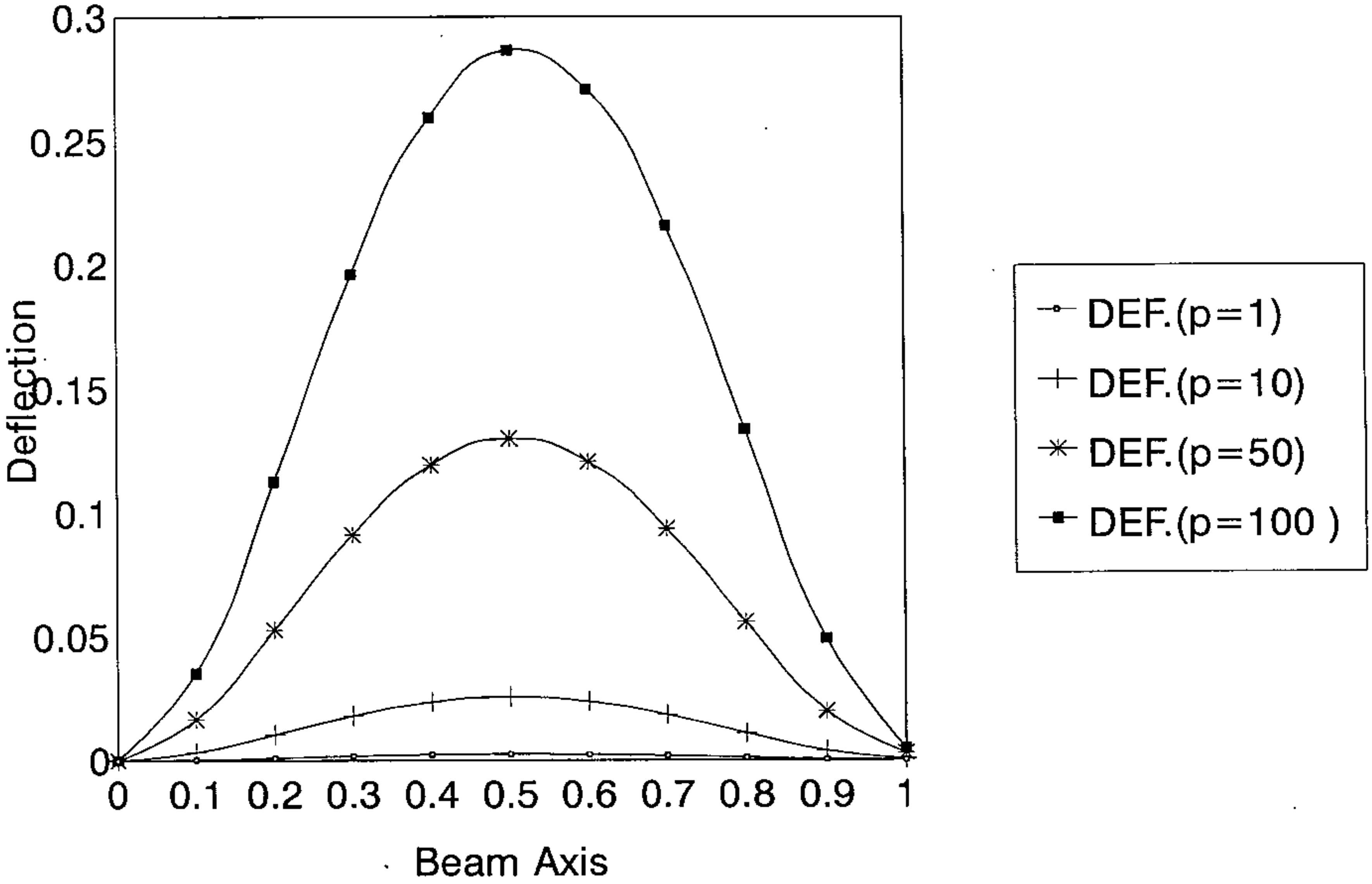


Fig.(5-5a) Deflection versus beam axis for clamped beam/
constant load with load variable,alfa=10,beta=10

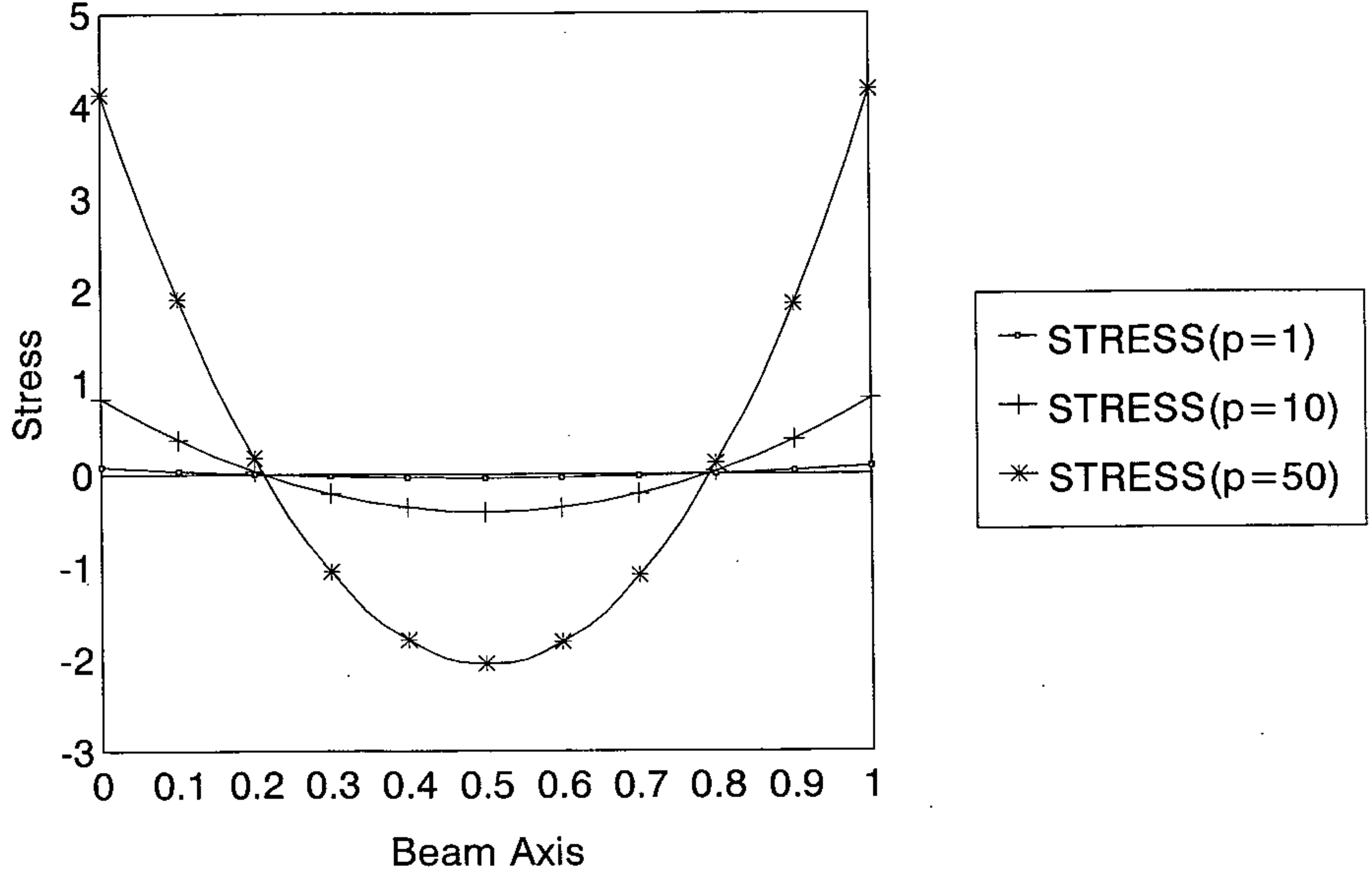


Fig.(5-5b) Stress versus beam axis for clamped beam/
constant load with load variable,alfa=10,beta=10

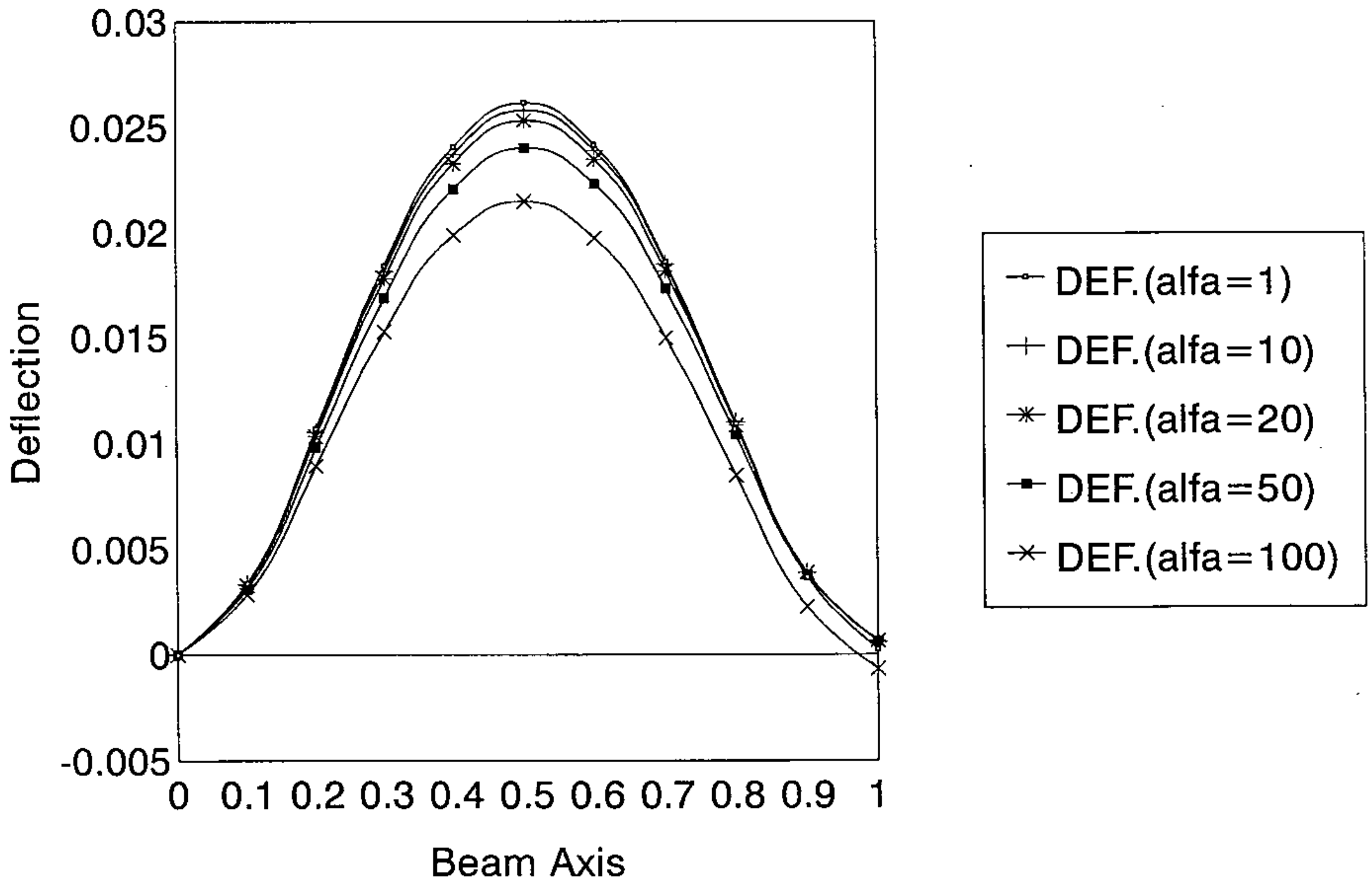


Fig.(5-6a) Deflection versus beam axis for clamped beam/ constant load with load=10, alfa=variable,beta=0

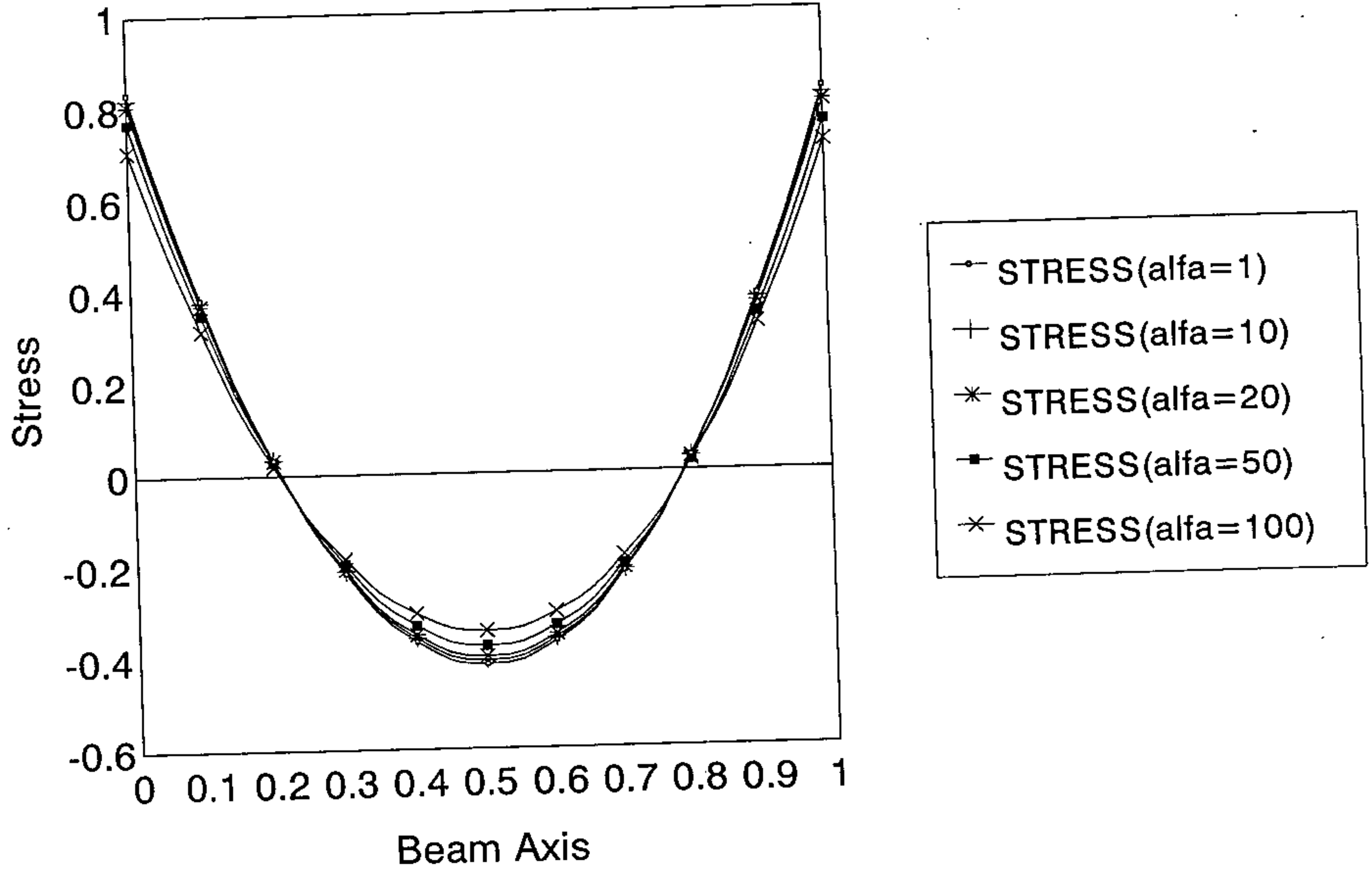


Fig.(5-6b) Stress versus beam axis for clamped beam/ constant load with load=10,alfa=variable,beta=0

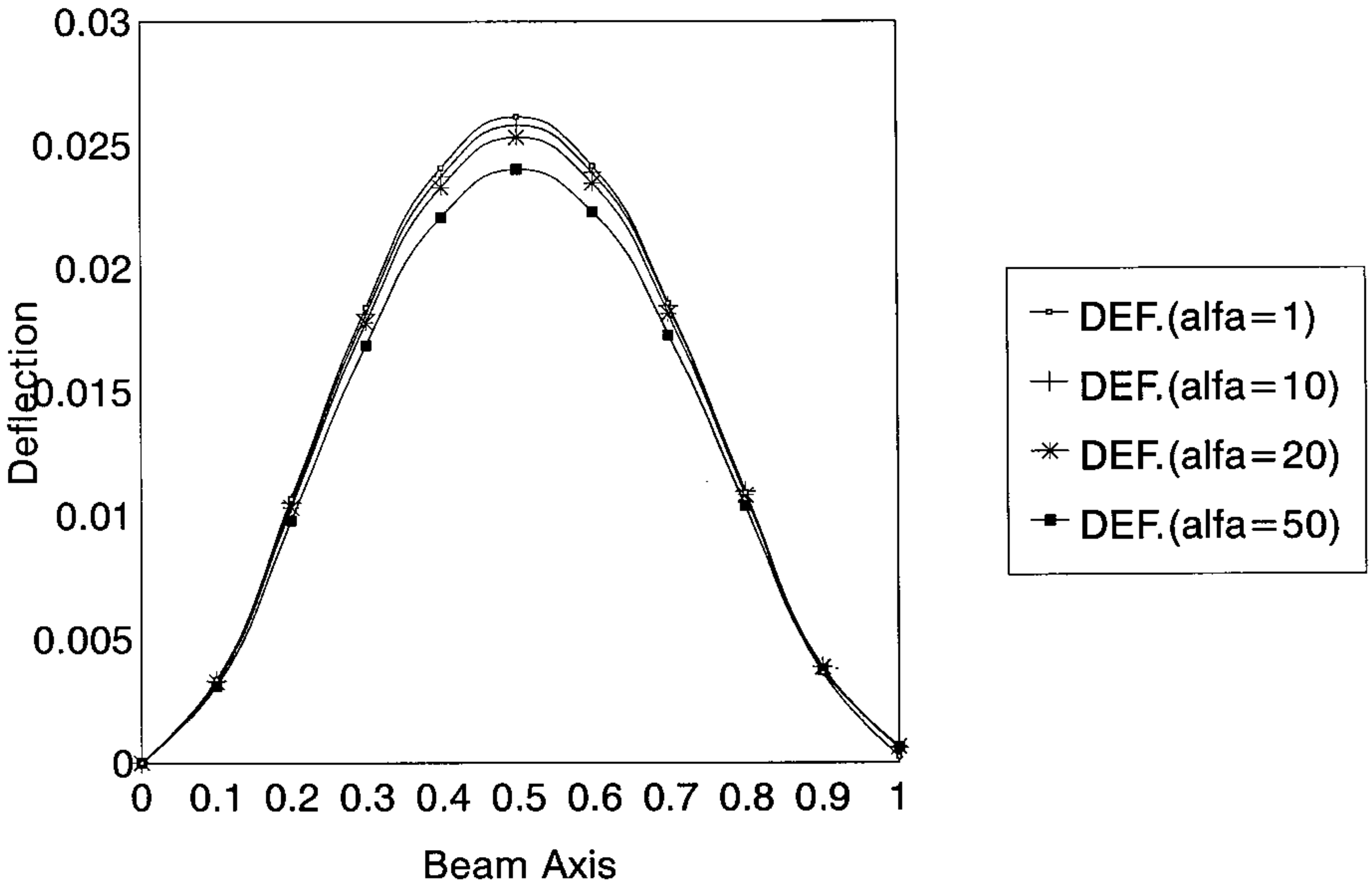


Fig.(5-7a)Deflection versus beam axis for clamped beam/ constant load with load=10, alfa=variable, beta=10

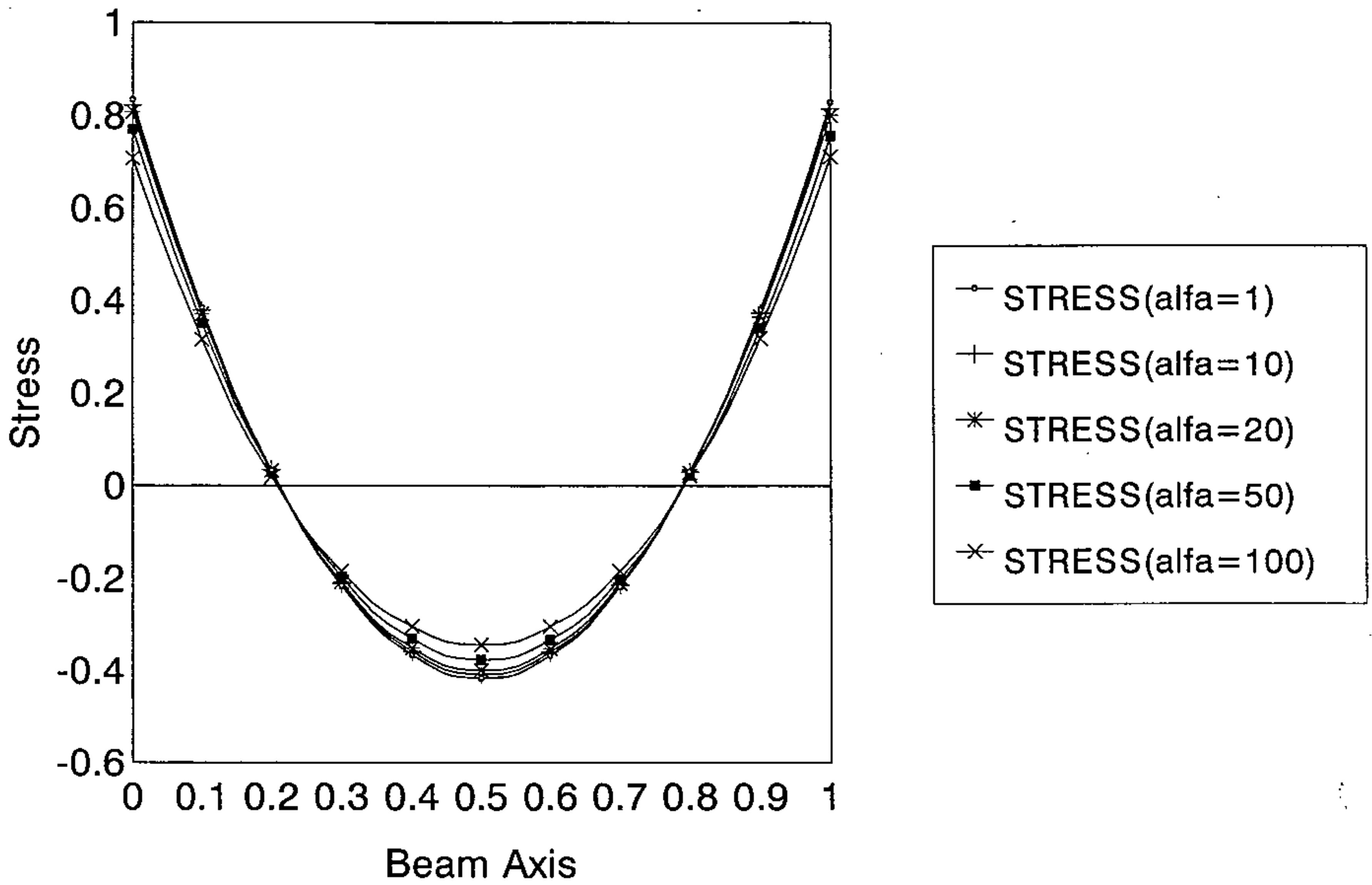


Fig.(5-7b) Stress versus beam axis for clamped beam/ constant load with load=10, alfa=variable, beta=10

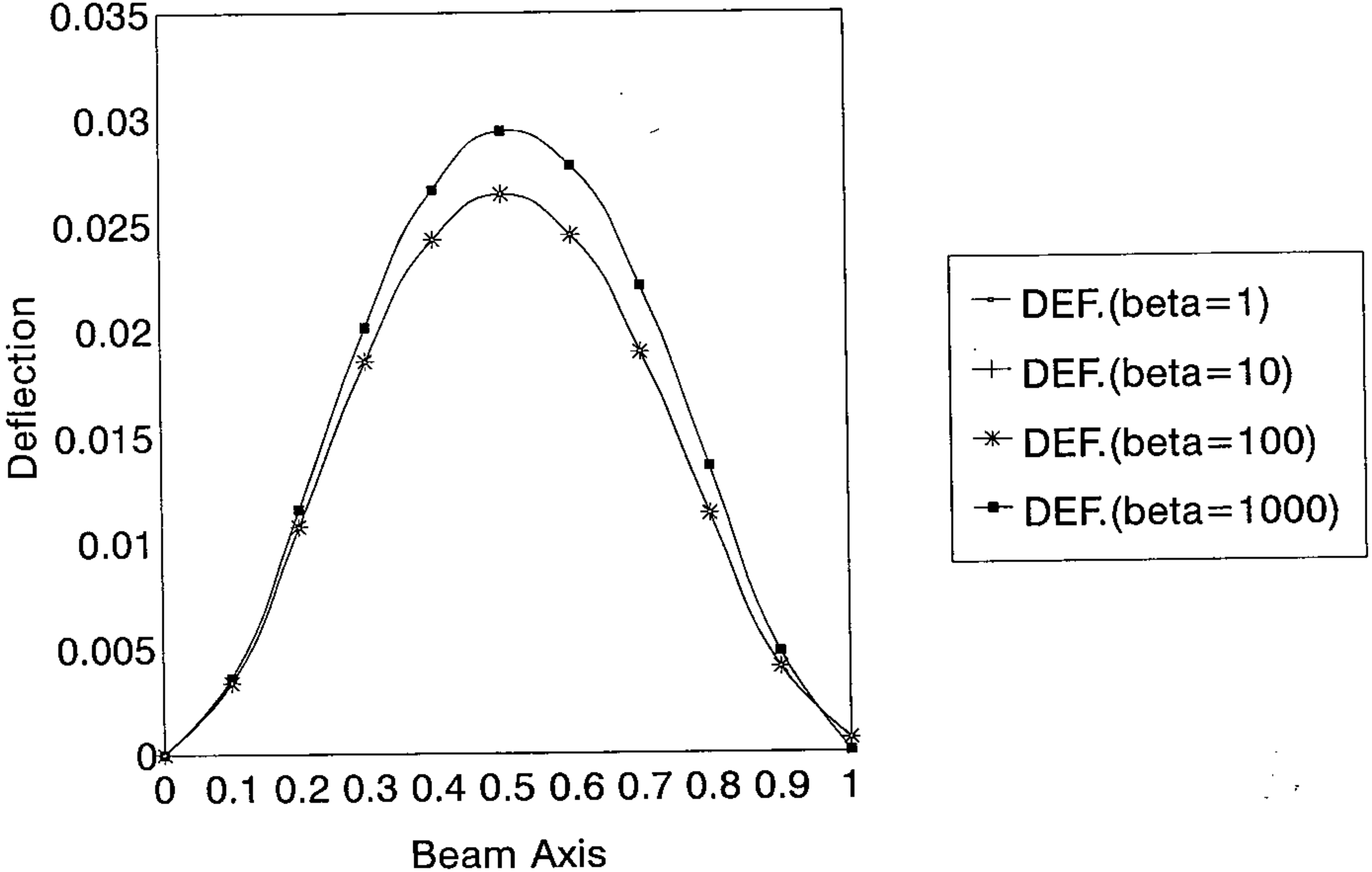


Fig.(5-8a) Deflection versus beam axis for clamped beam/ constant load with load=10, alfa=0 ,beta=variable

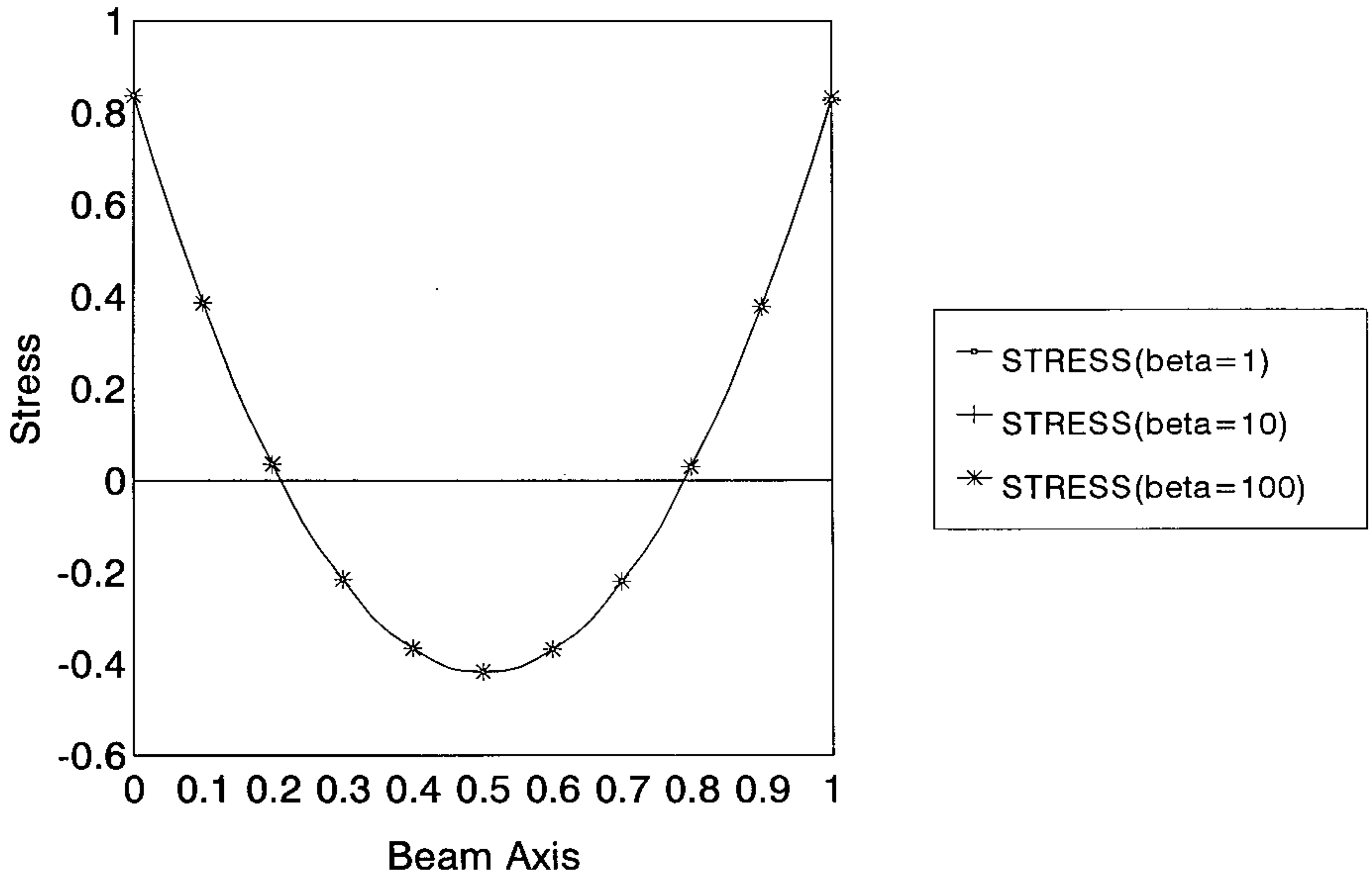


Fig.(5-8b) Stress versus beam axis for clamped beam/ constant load with load=10, alfa=0 ,beta=variable

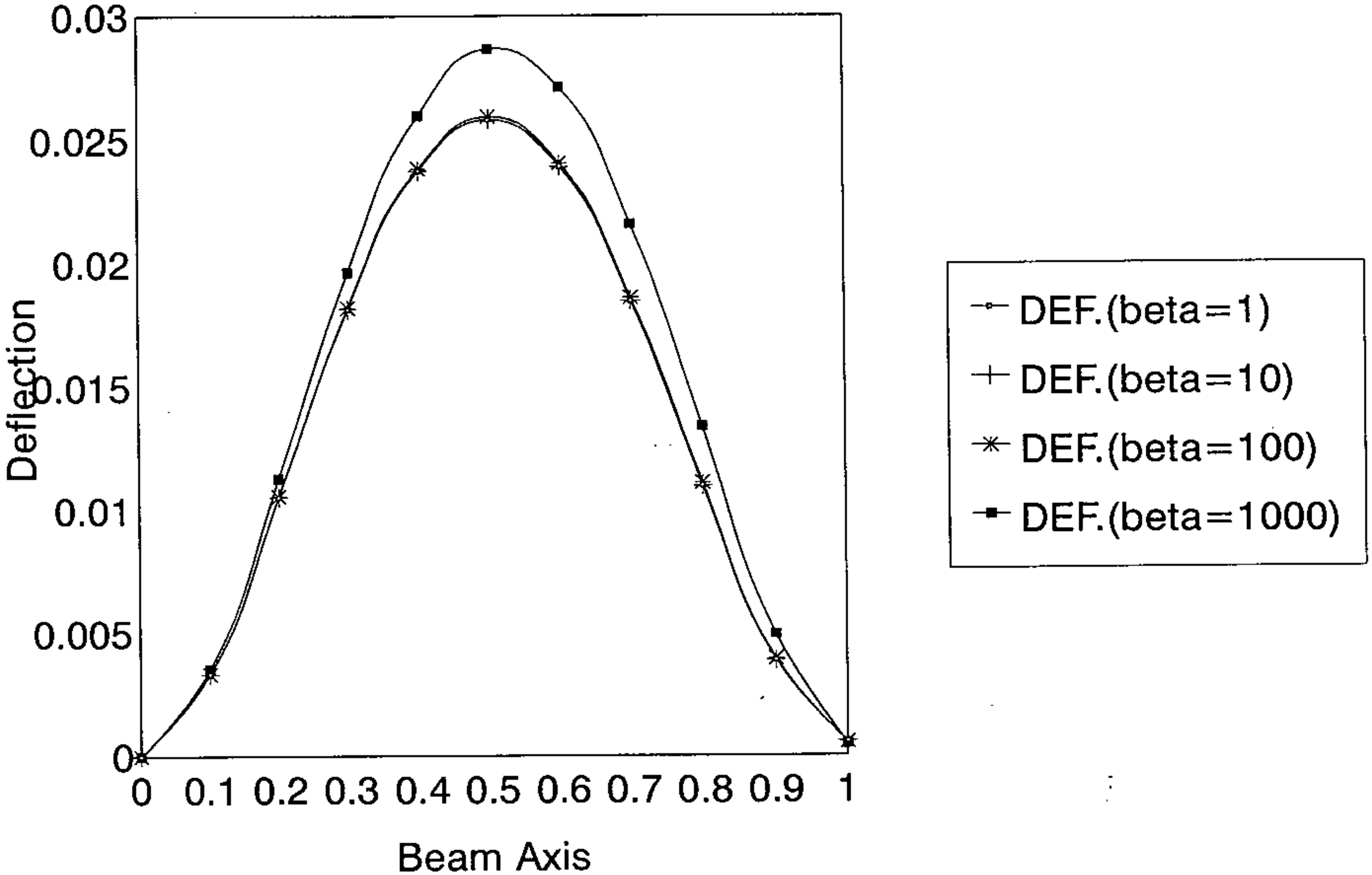


Fig.(5-9a) Deflection versus beam axis for clamped beam/ constant load with load=10, alfa=10 ,beta=variable

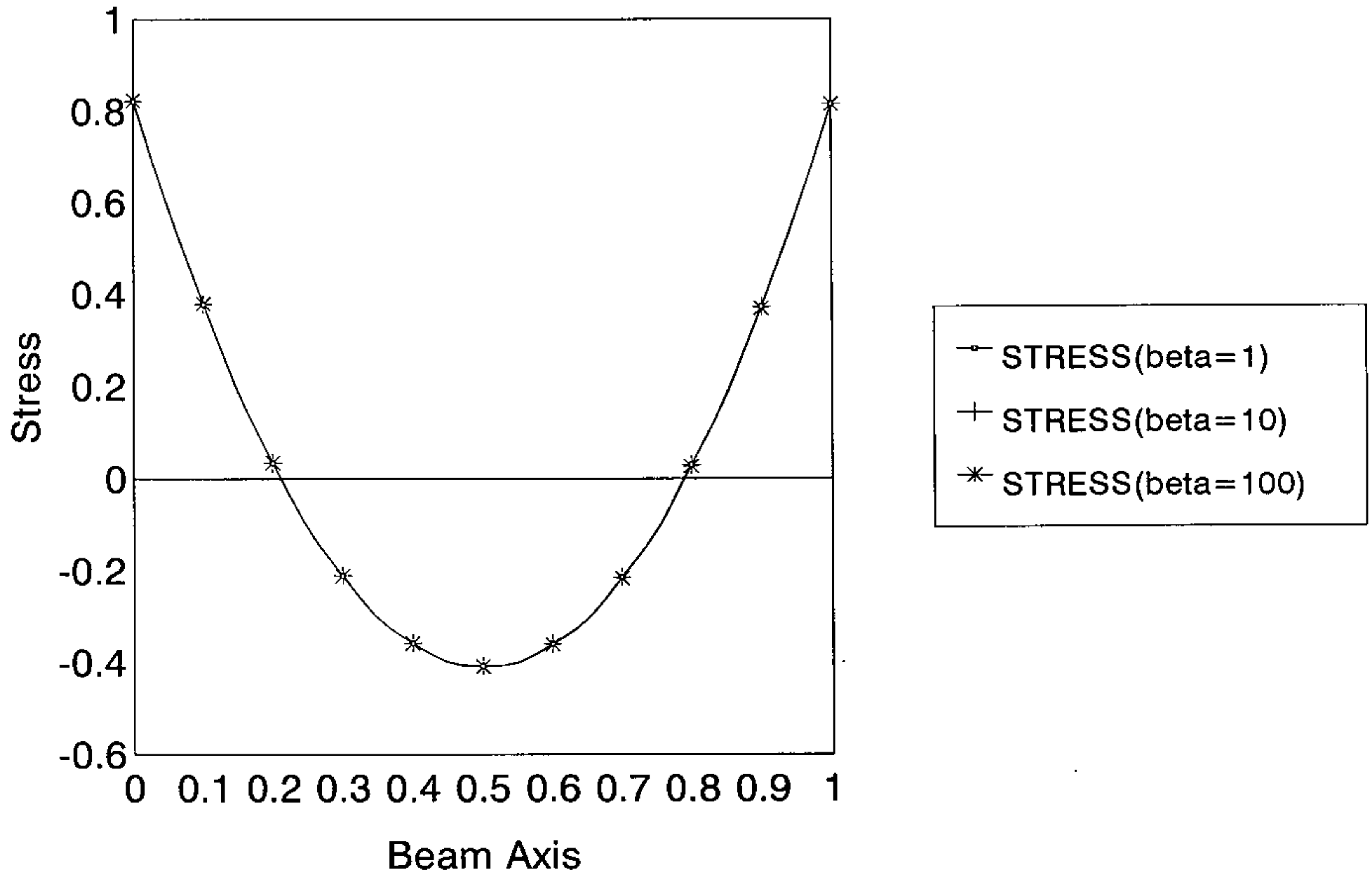


Fig.(5-9b) Stress versus beam axis for clamped beam/ constant load with load=10, alfa=10 ,beta=variable

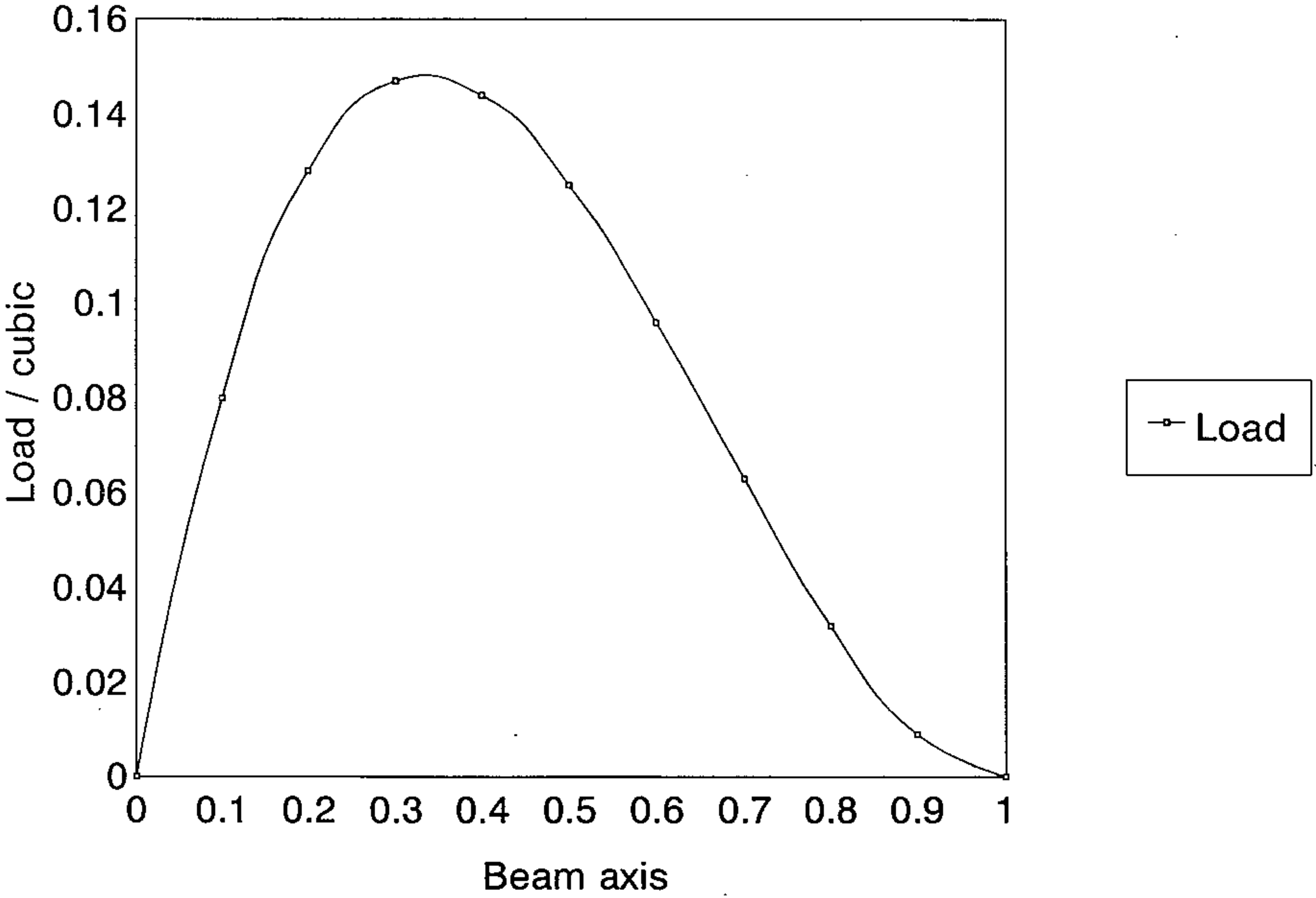


Fig.(5-10) Cubic load (X**3 - 2X**2 + X) (unsymmetric).

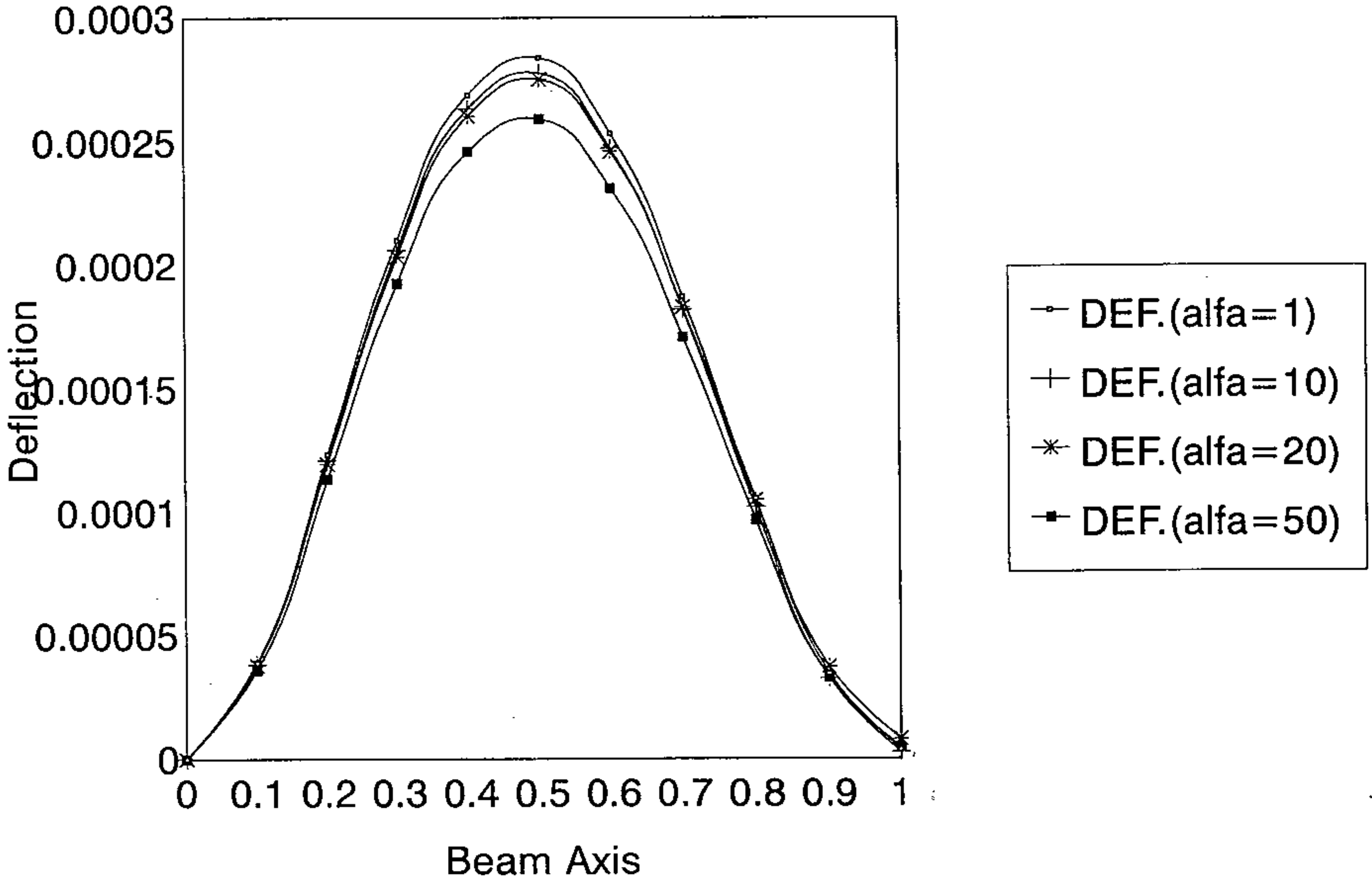


Fig.(5-11a) Deflection versus beam axis for clamped beam/
cubic load with alfa variable, beta=0

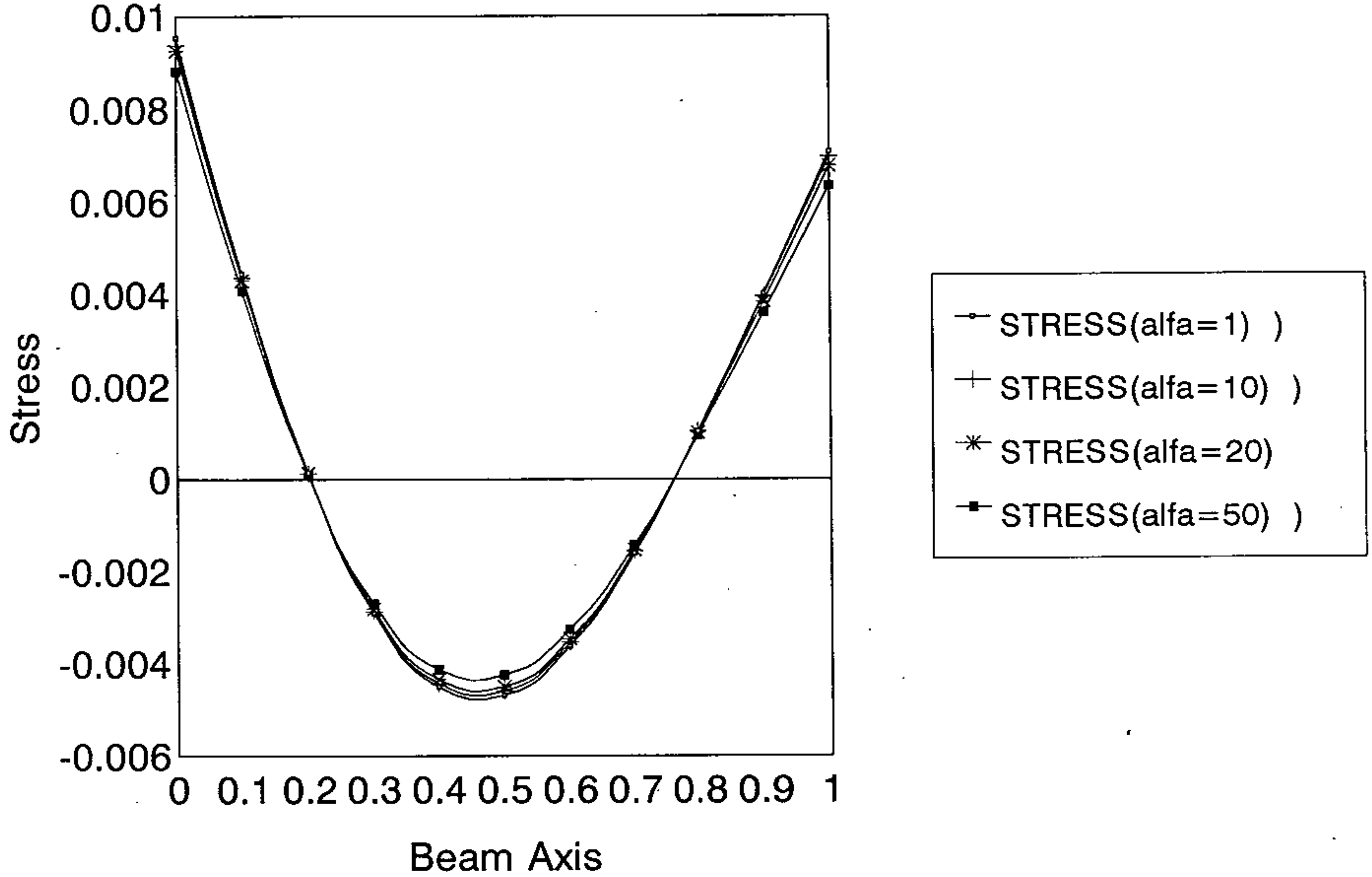


Fig.(5-11b) Stress versus beam axis for clamped beam/ cubic load with alfa variable, beta=0

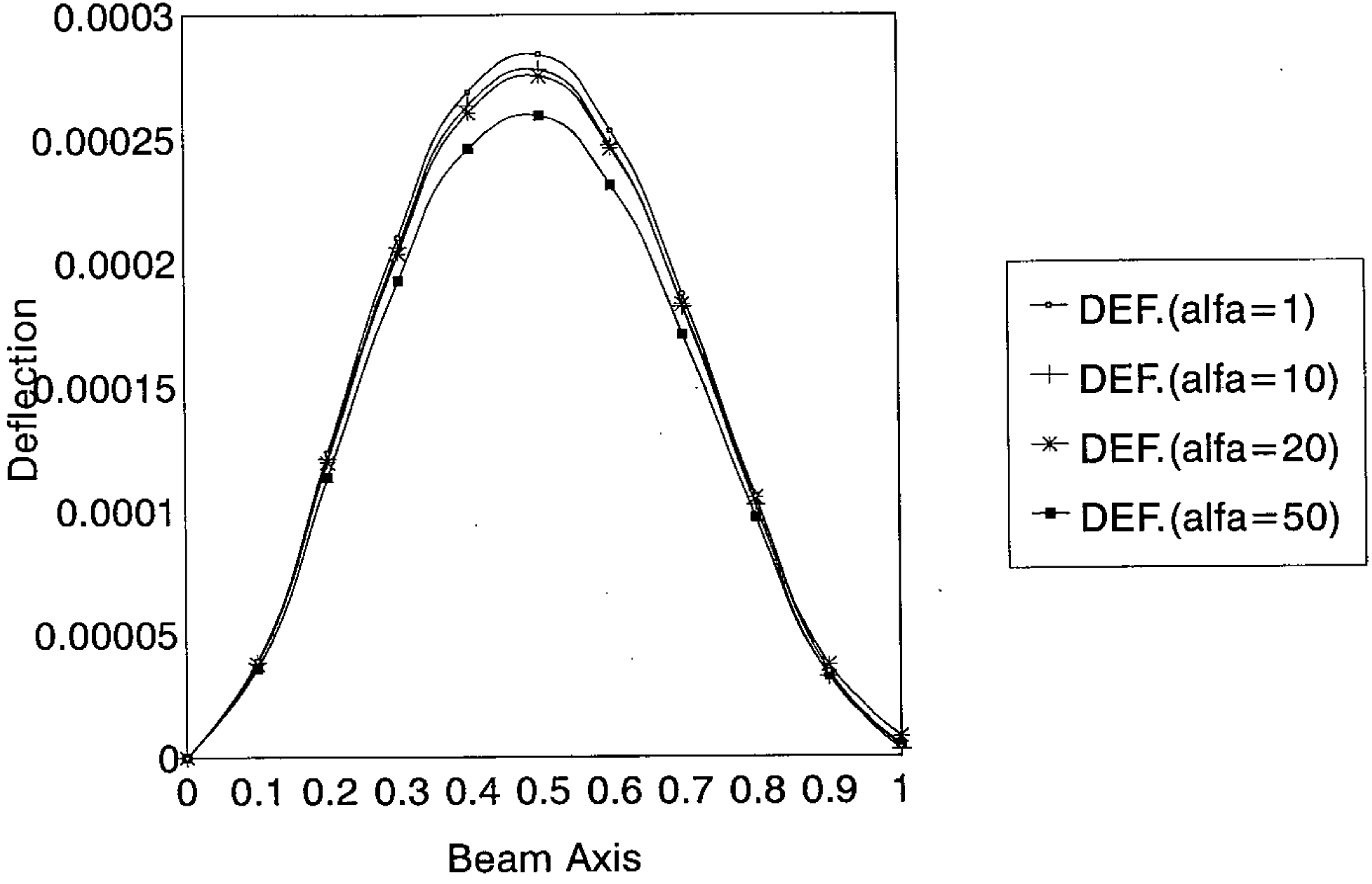


Fig.(5-12a) Deflection versus beam axis for clamped beam/ cubic load with alfa variable ,beta=10

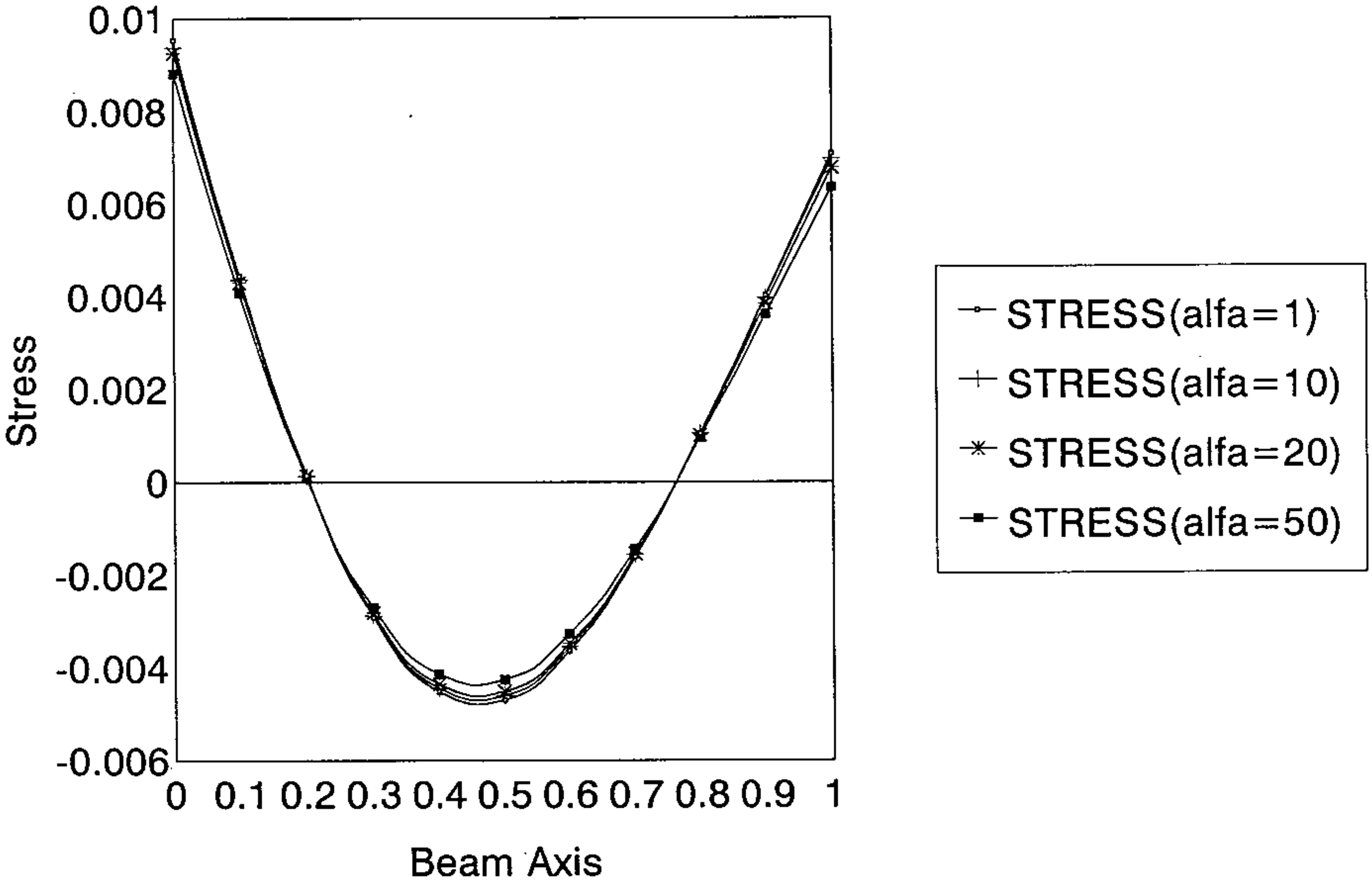


Fig.(5-12b) Stress versus beam axis for clamped beam/ cubic load with alfa variable, beta=10

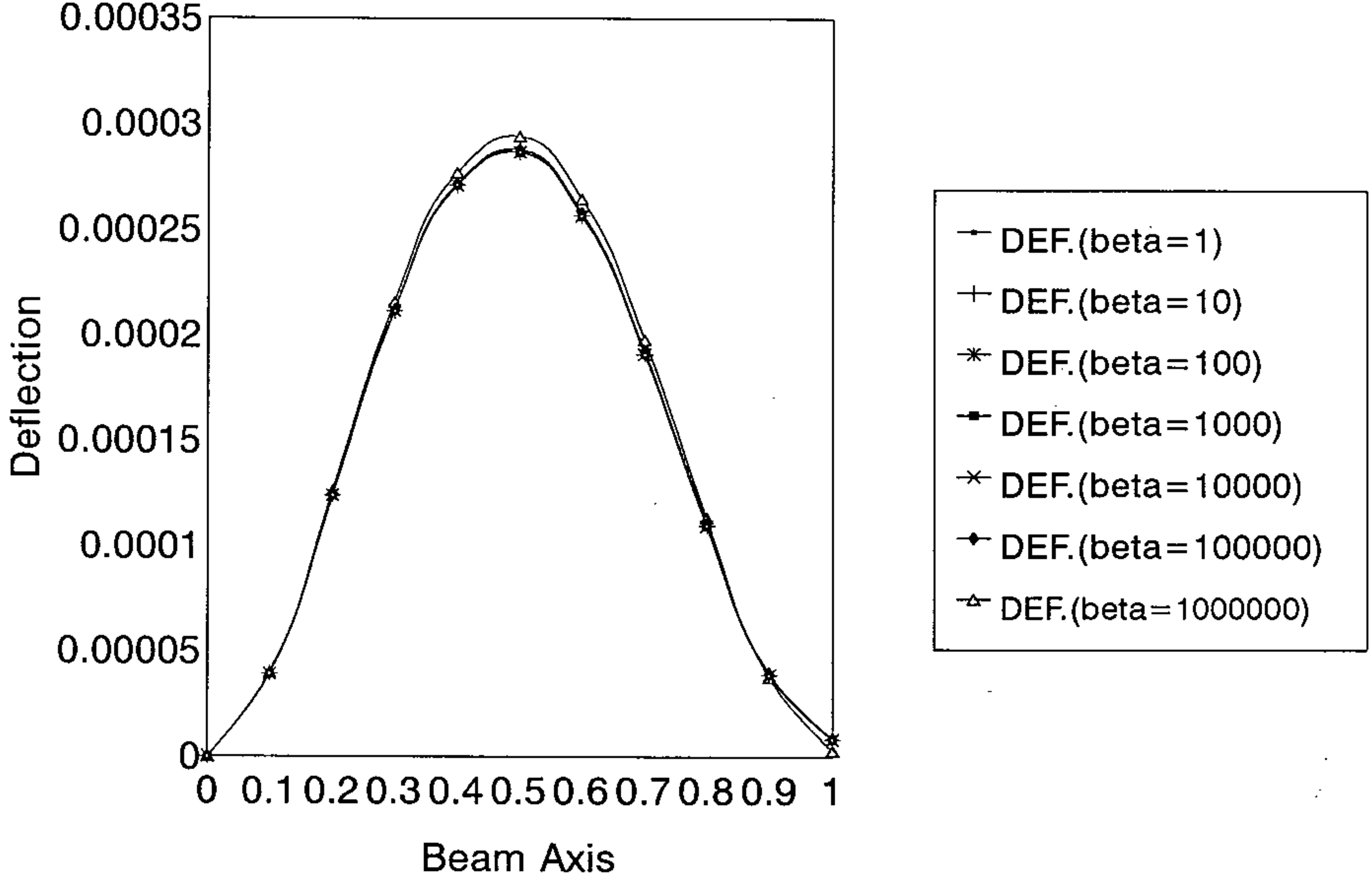


Fig.(5-13a) Deflection versus beam axis for clamped beam/ cubic load with alfa=0, beta variable

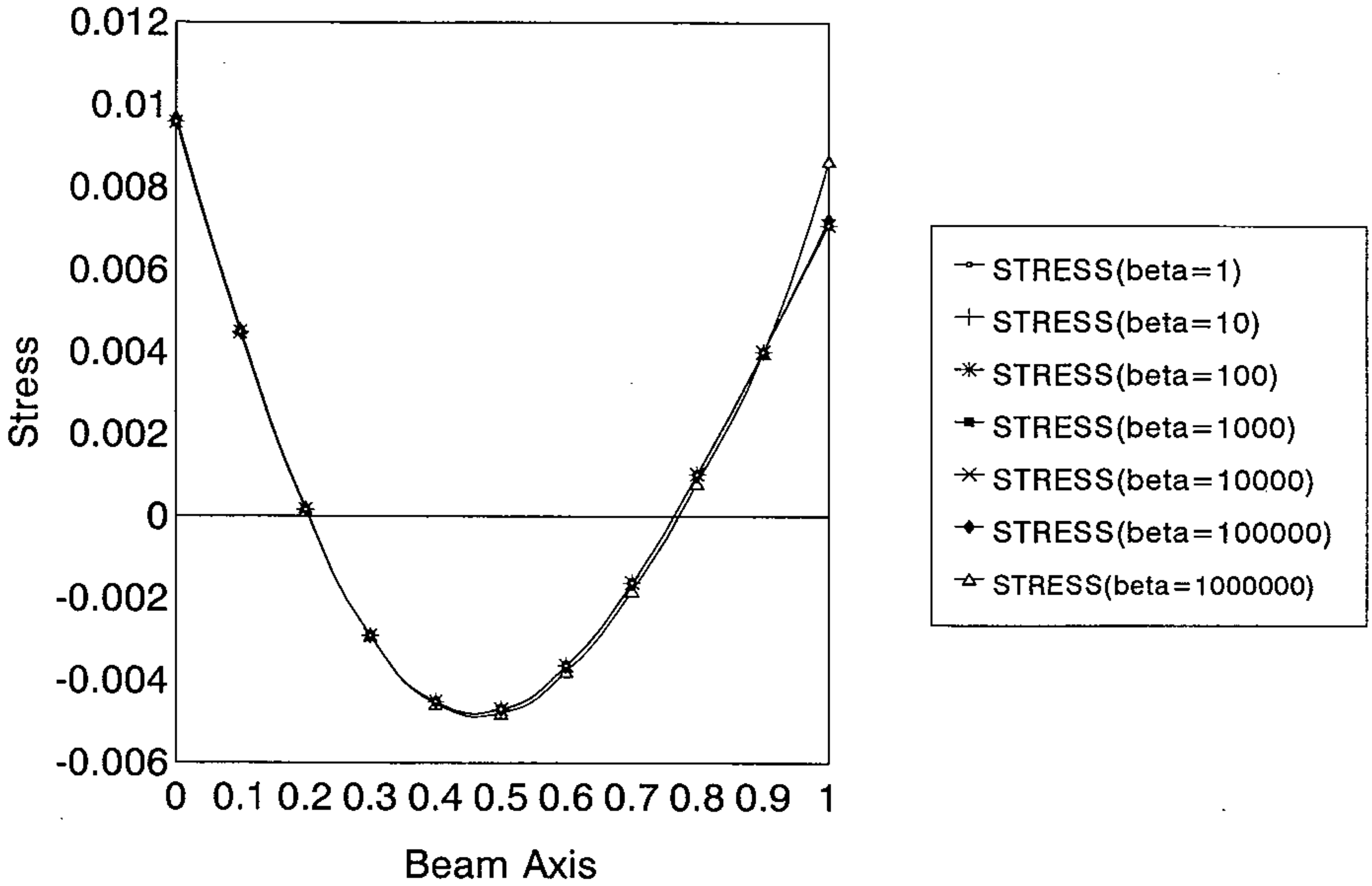


Fig.(5-13b) Stress versus beam axis for clamped beam/ cubic load with alfa=0 ,beta variable

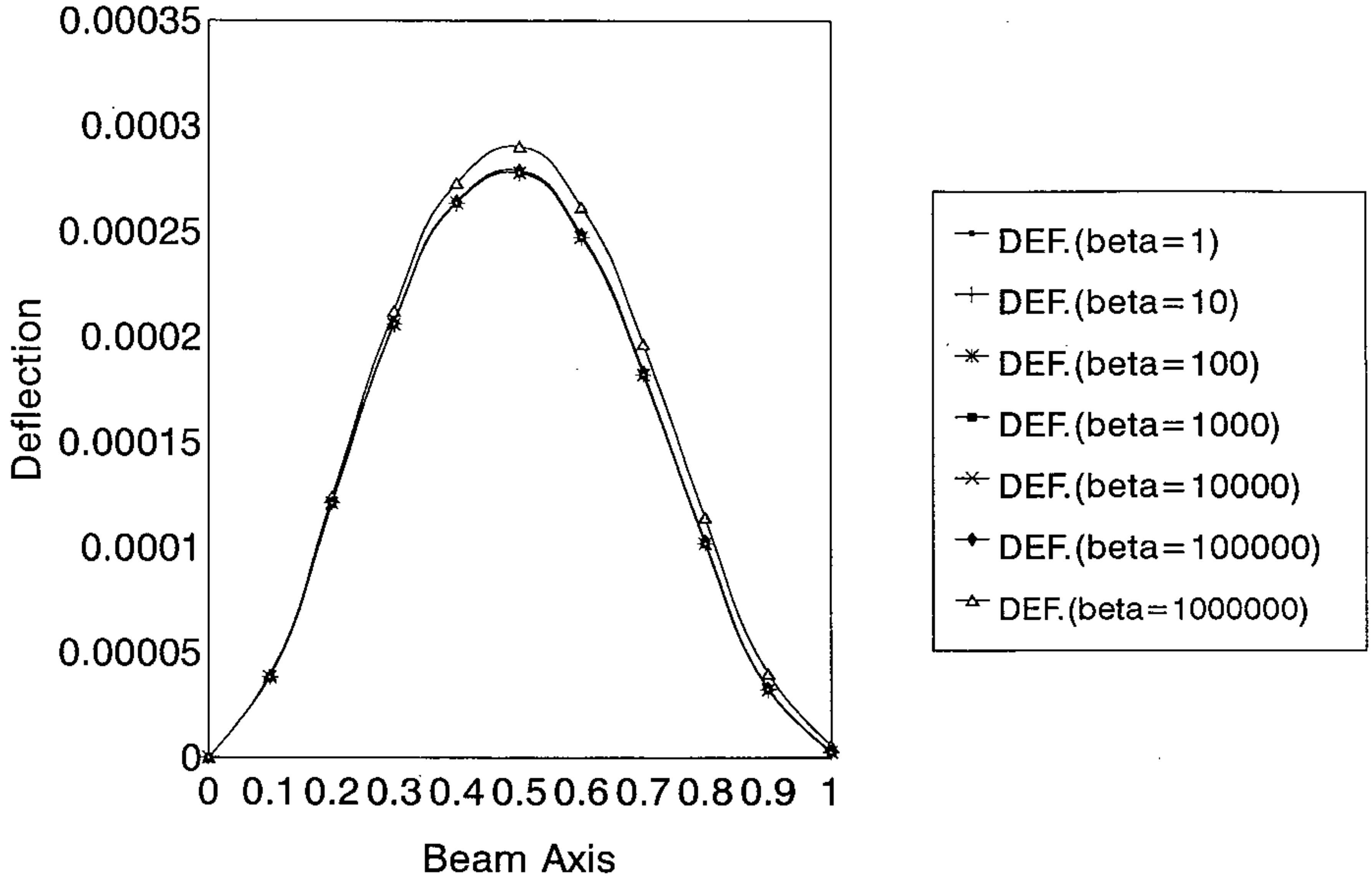


Fig.(5-14a) Deflection versus beam axis for clamped beam/ cubic load with alfa=10, beta variable

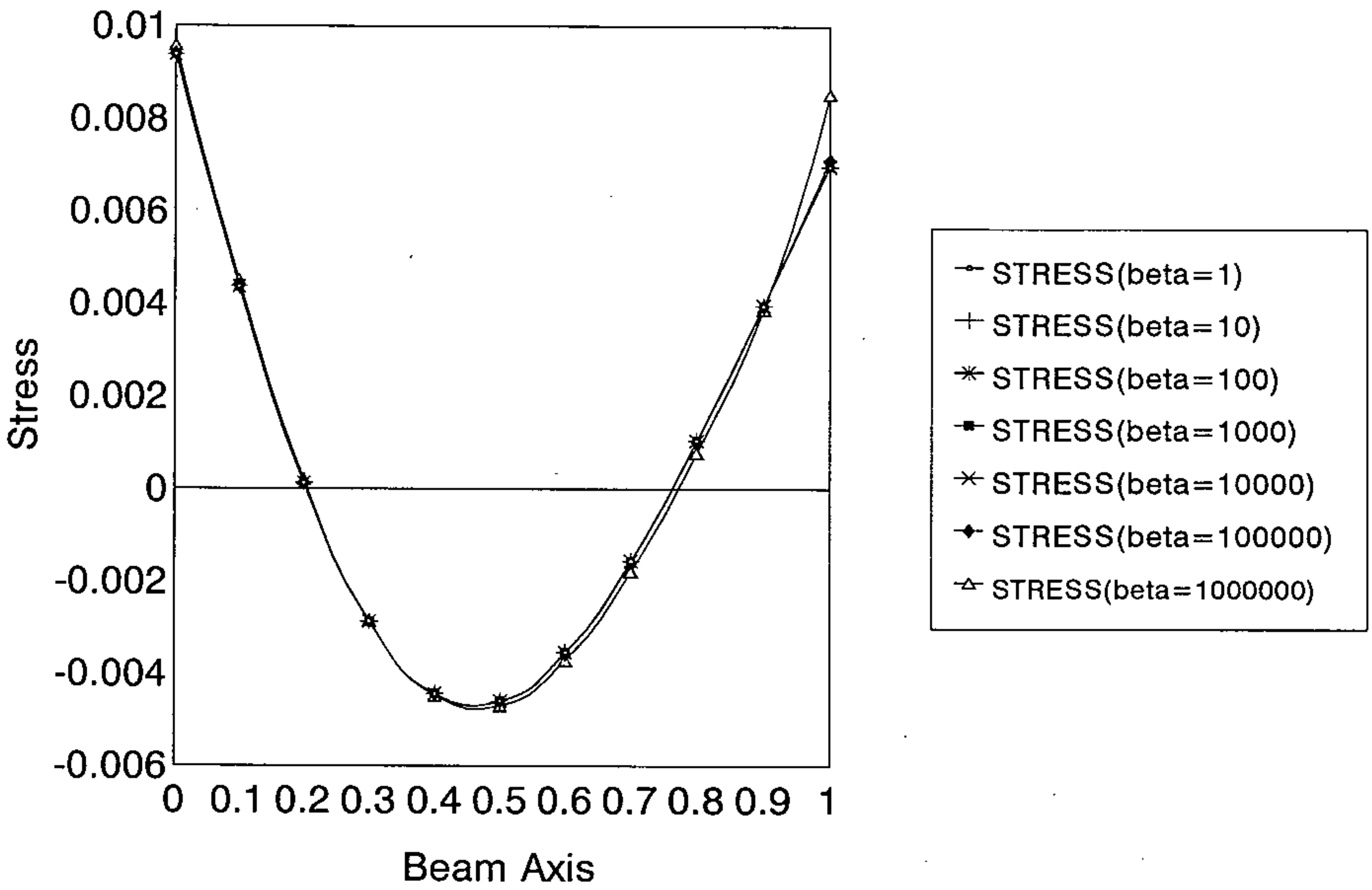


Fig.(5-14b) Stress versus beam axis for clamped beam/ cubic load with alfa=10, beta variable

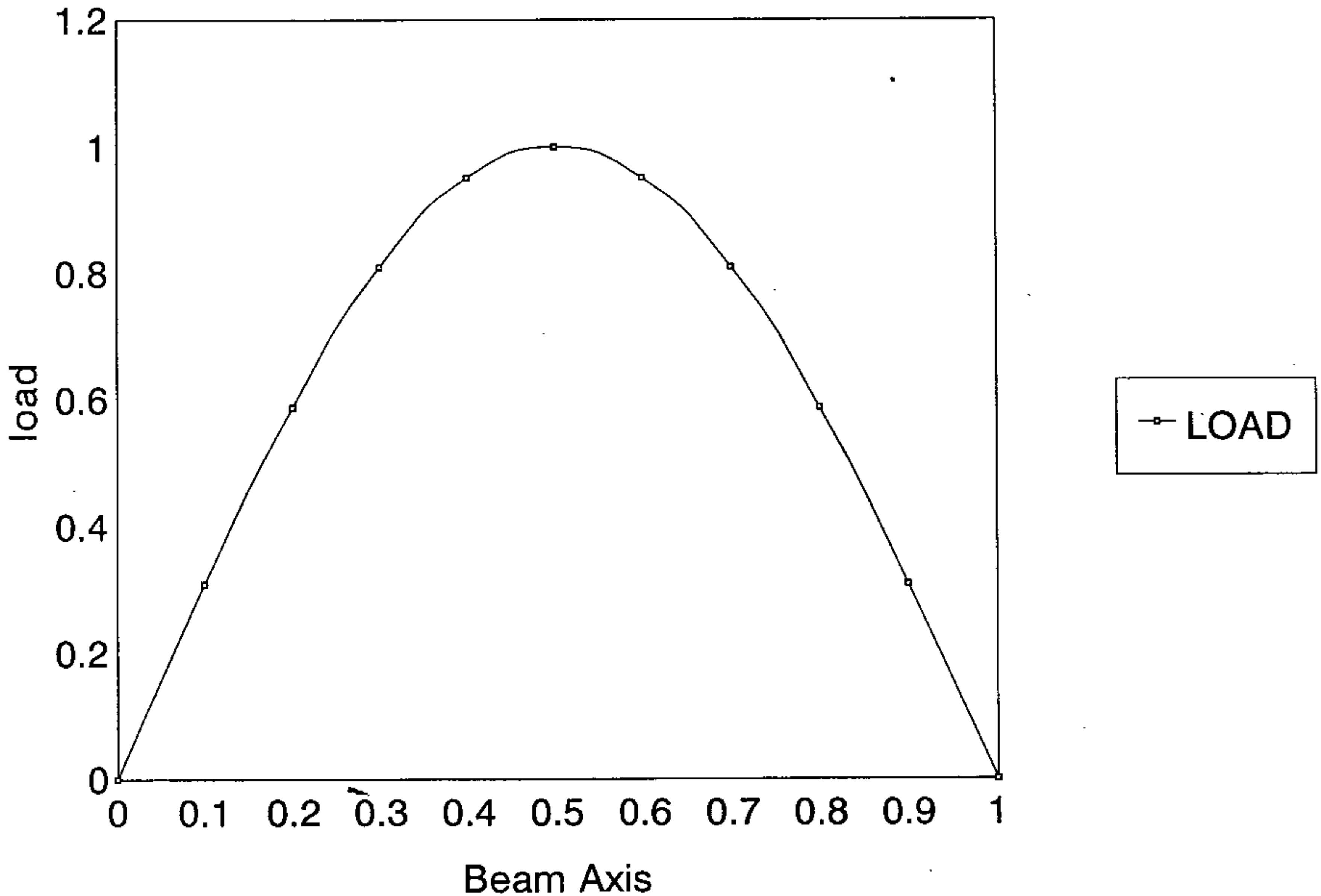


Fig.(5-15) Sinusoidal load (sin 22/7 X) (symmetric)

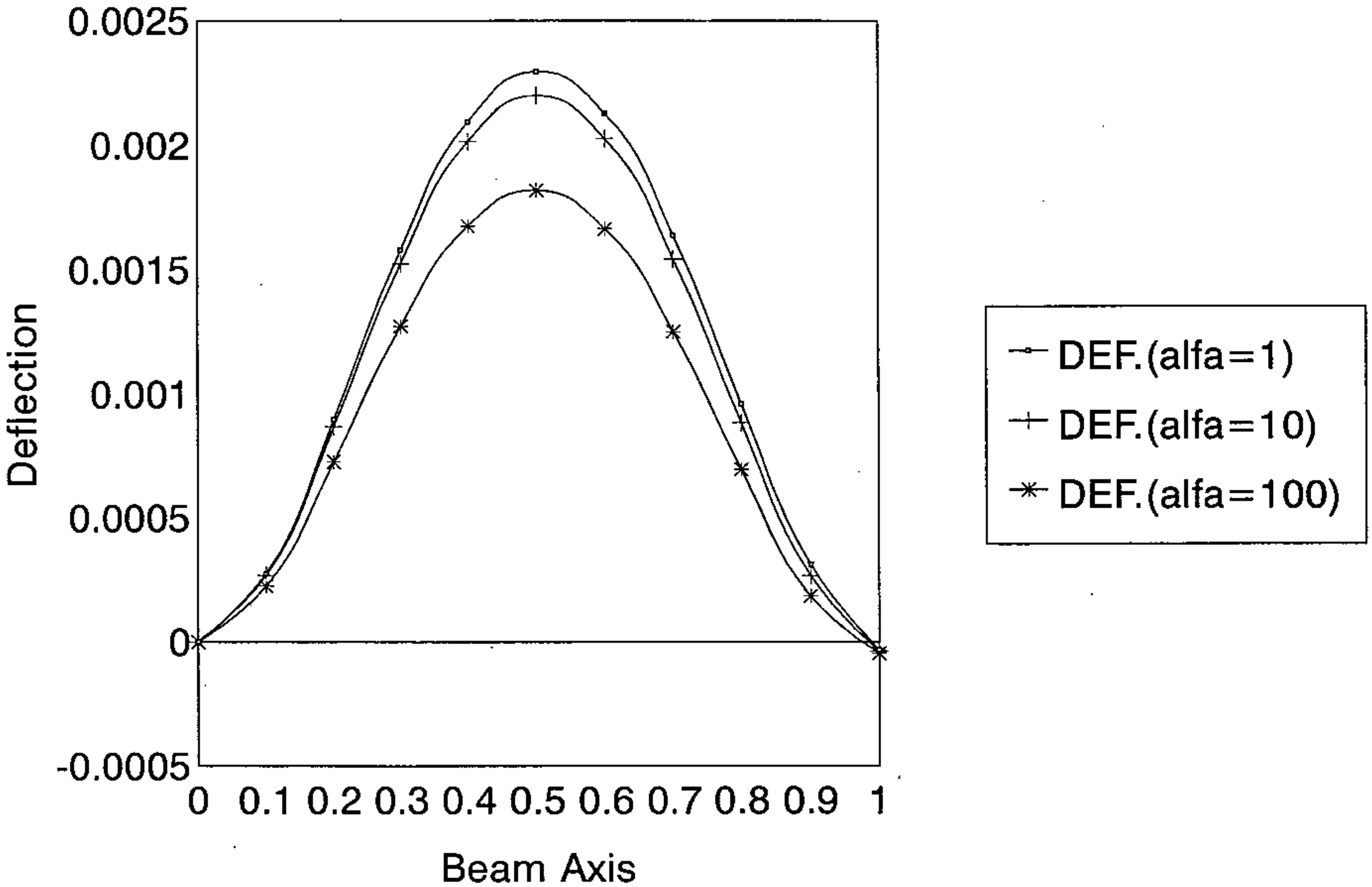


Fig.(5-16a)Deflection versus beam axis for clamped beam/ sinusoidal load with alfa variable ,beta=0

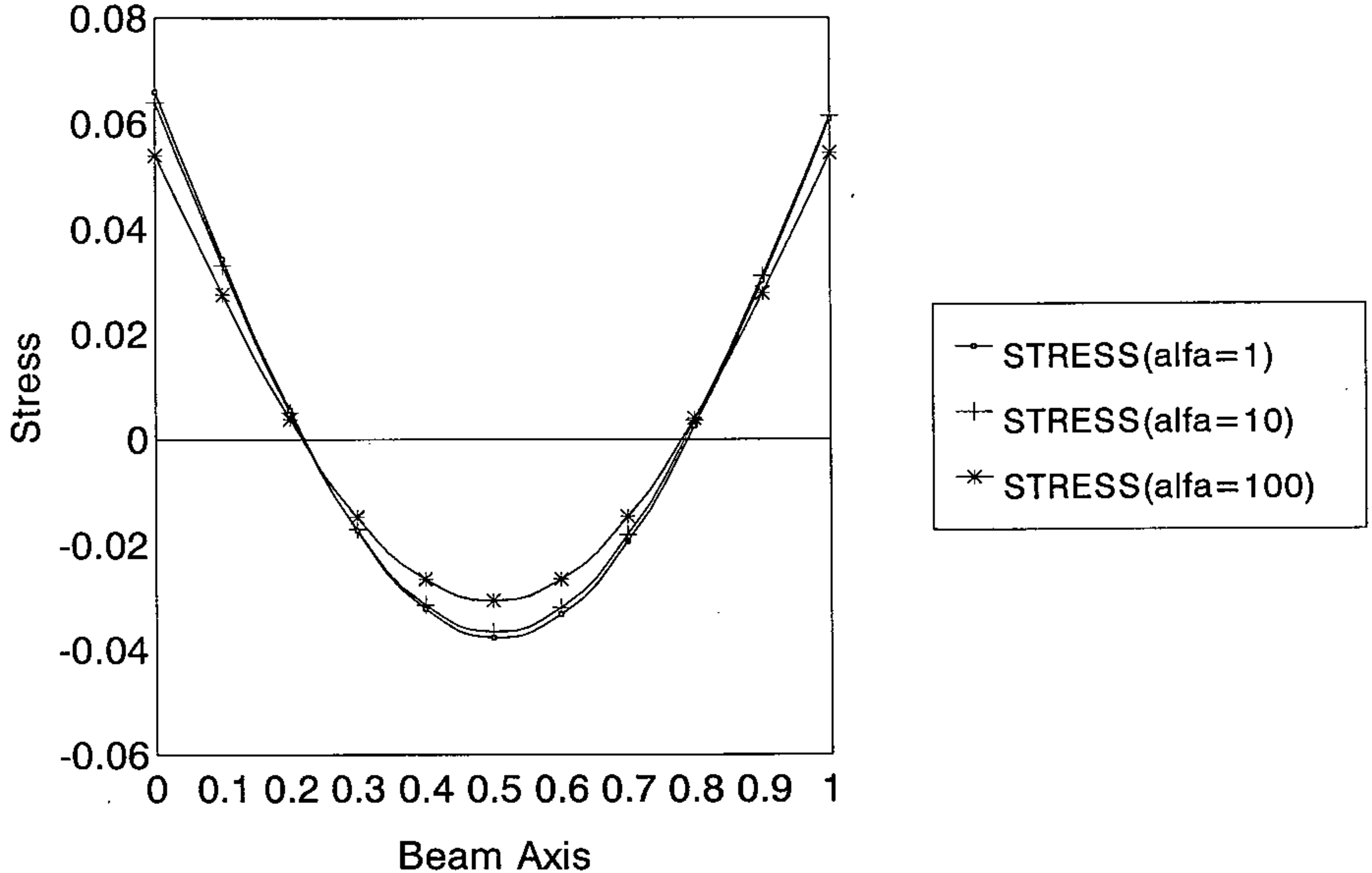


Fig.(5-16b)Stress versus beam axis for clamped beam/ sinusoidal load with alfa variable,beta=0

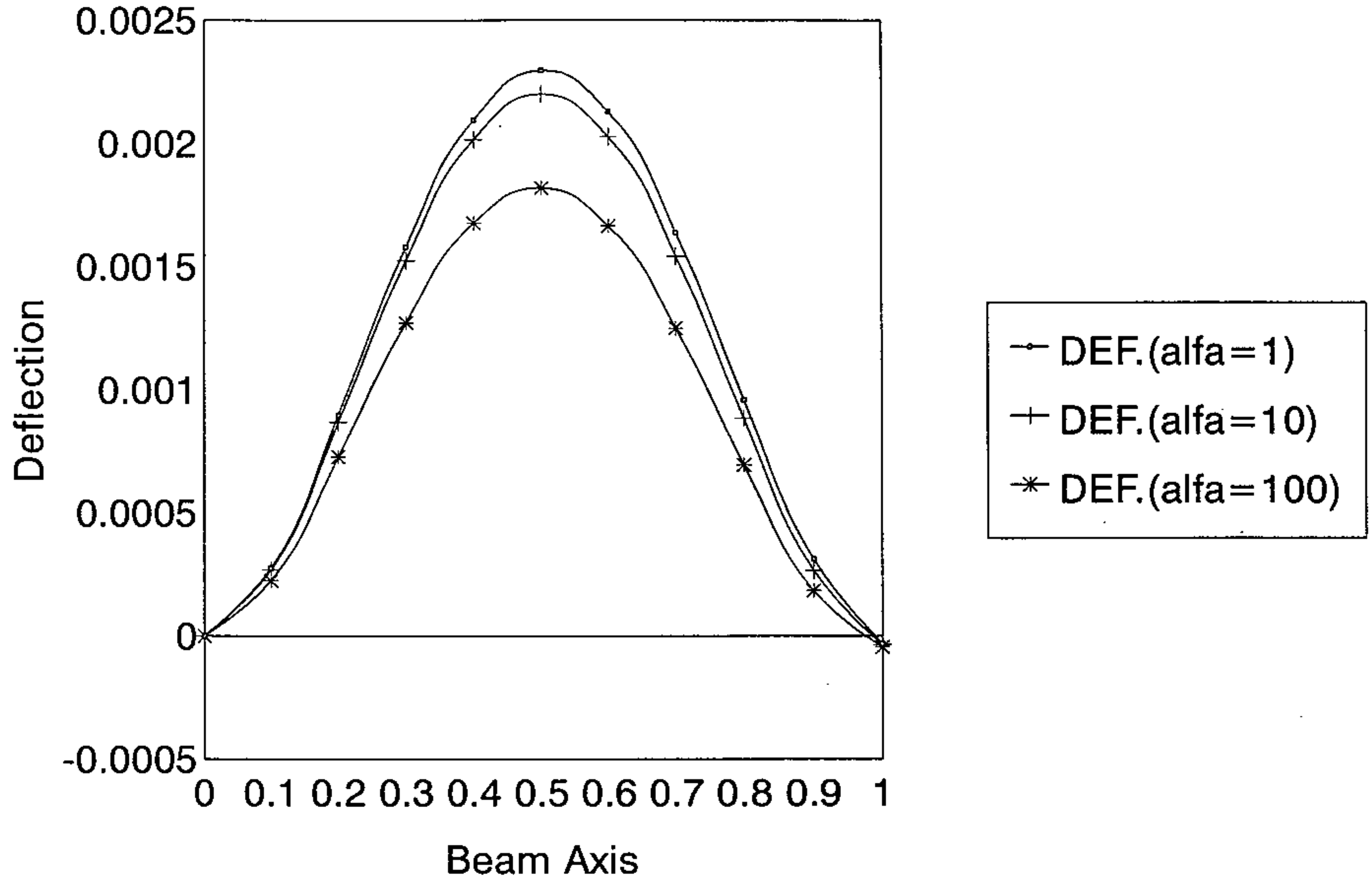


Fig.(5-17a)Deflection versus beam axis for clamped beam/ sinusoidal load with alfa variable ,beta=10

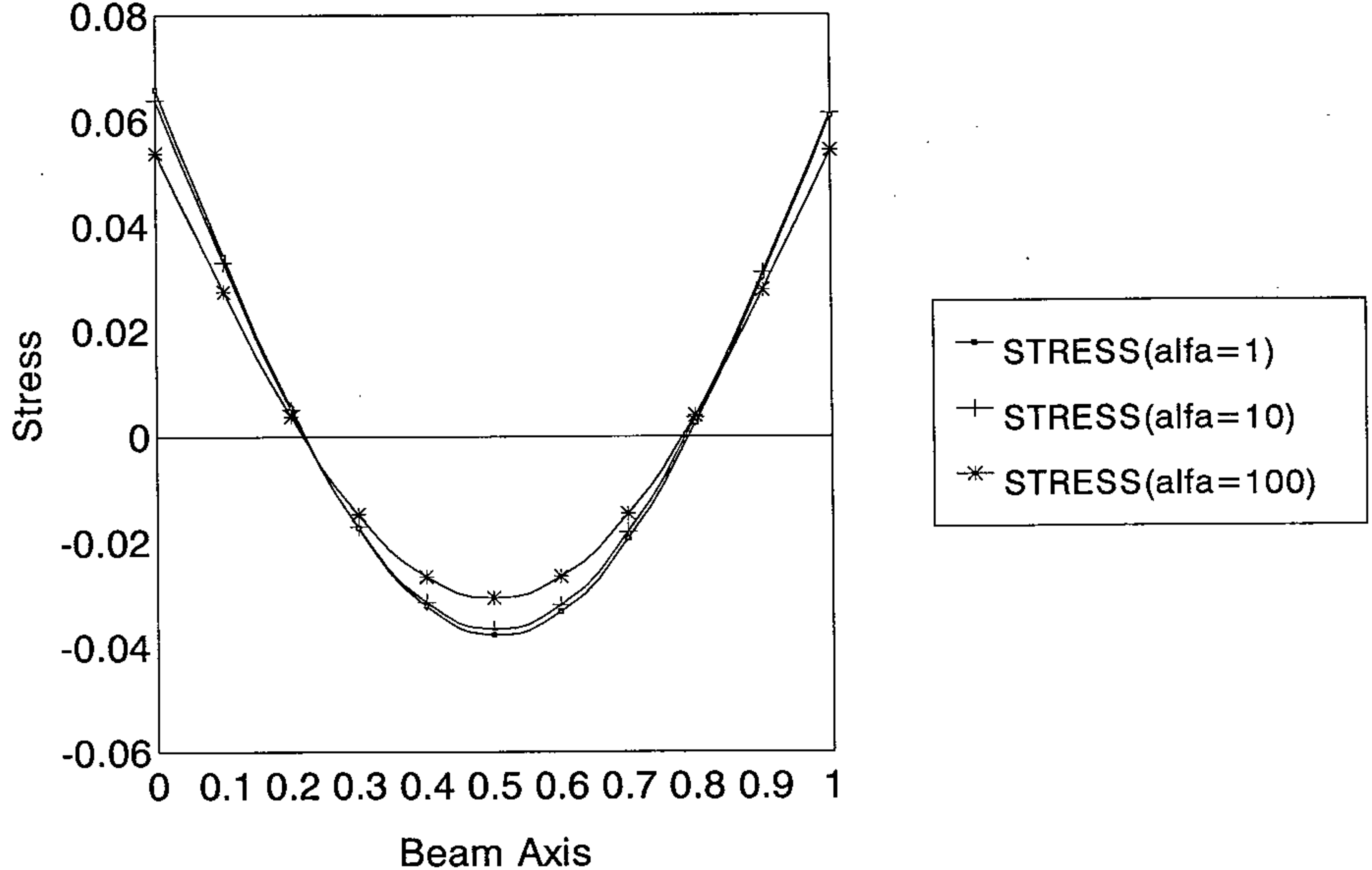


Fig.(5-17b)Stress versus beam axis for clamped beam/ sinusoidal load with alfa variable ,beta=10

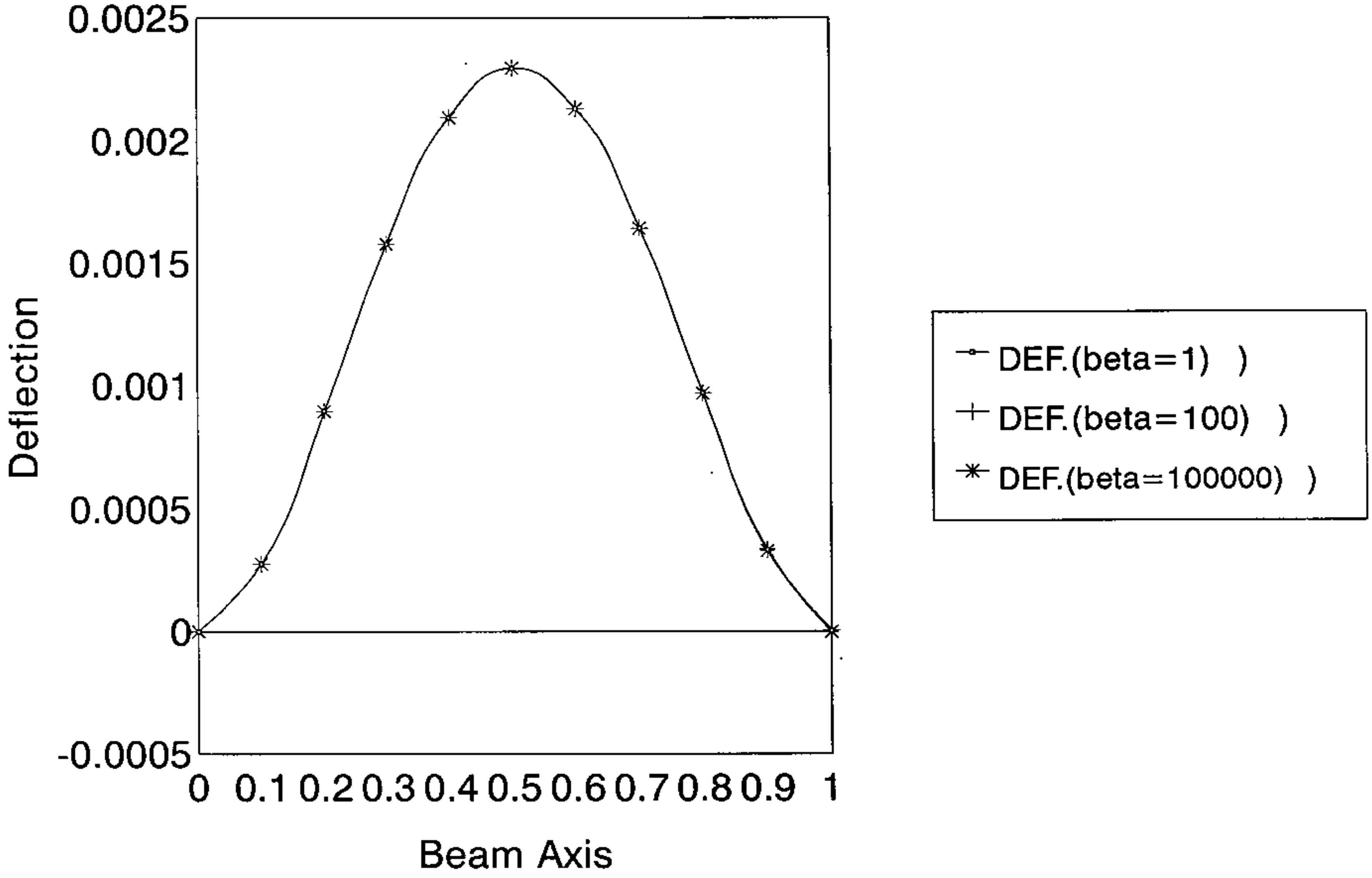


Fig.(5-18a)Deflection versus beam axis for clamped beam/ sinusoidal load with alfa=0 ,beta variable

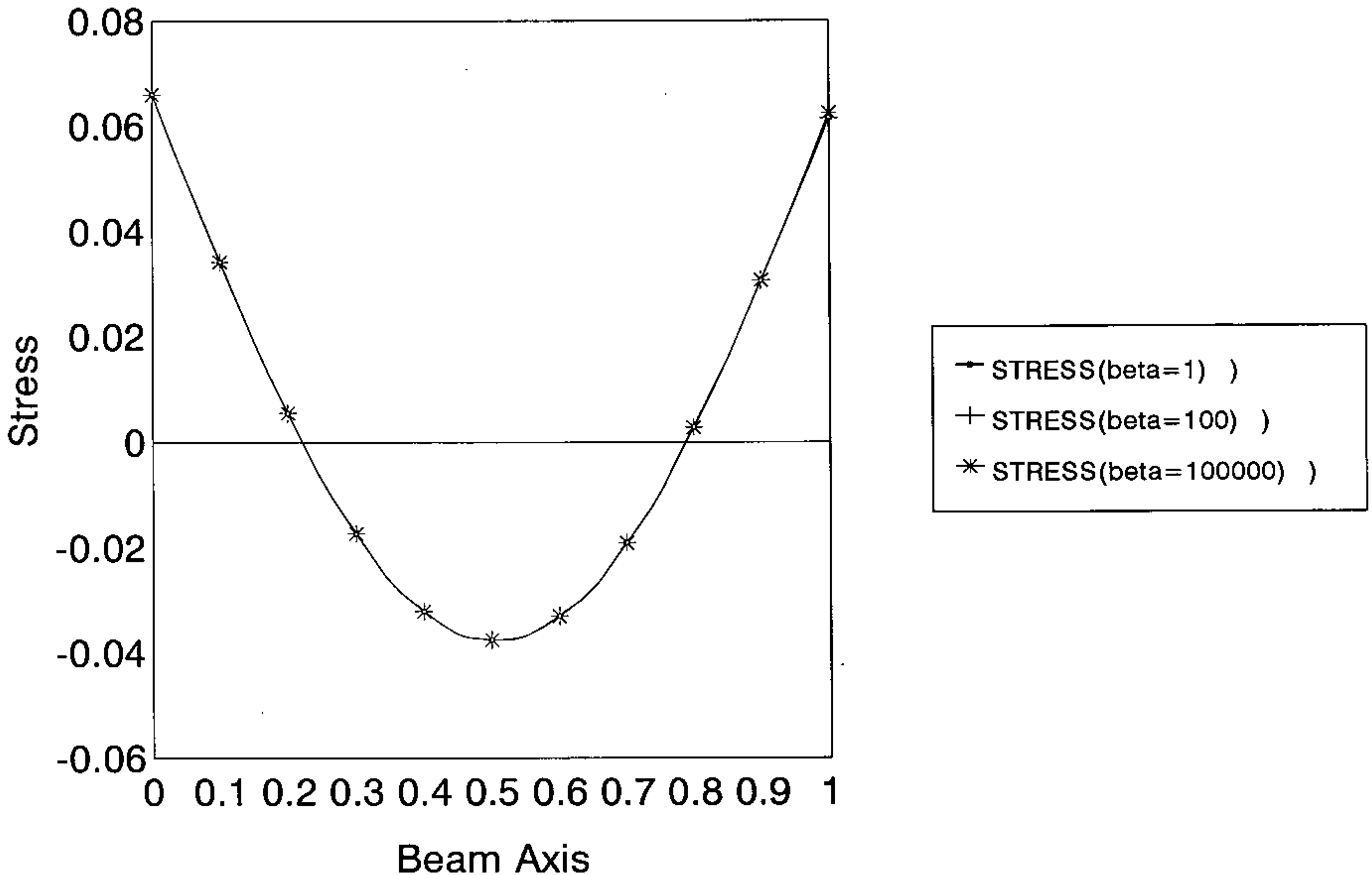


Fig.(5-18b)Stress versus beam axis for clamped beam/ sinusoidal load with alfa=0 ,beta variable

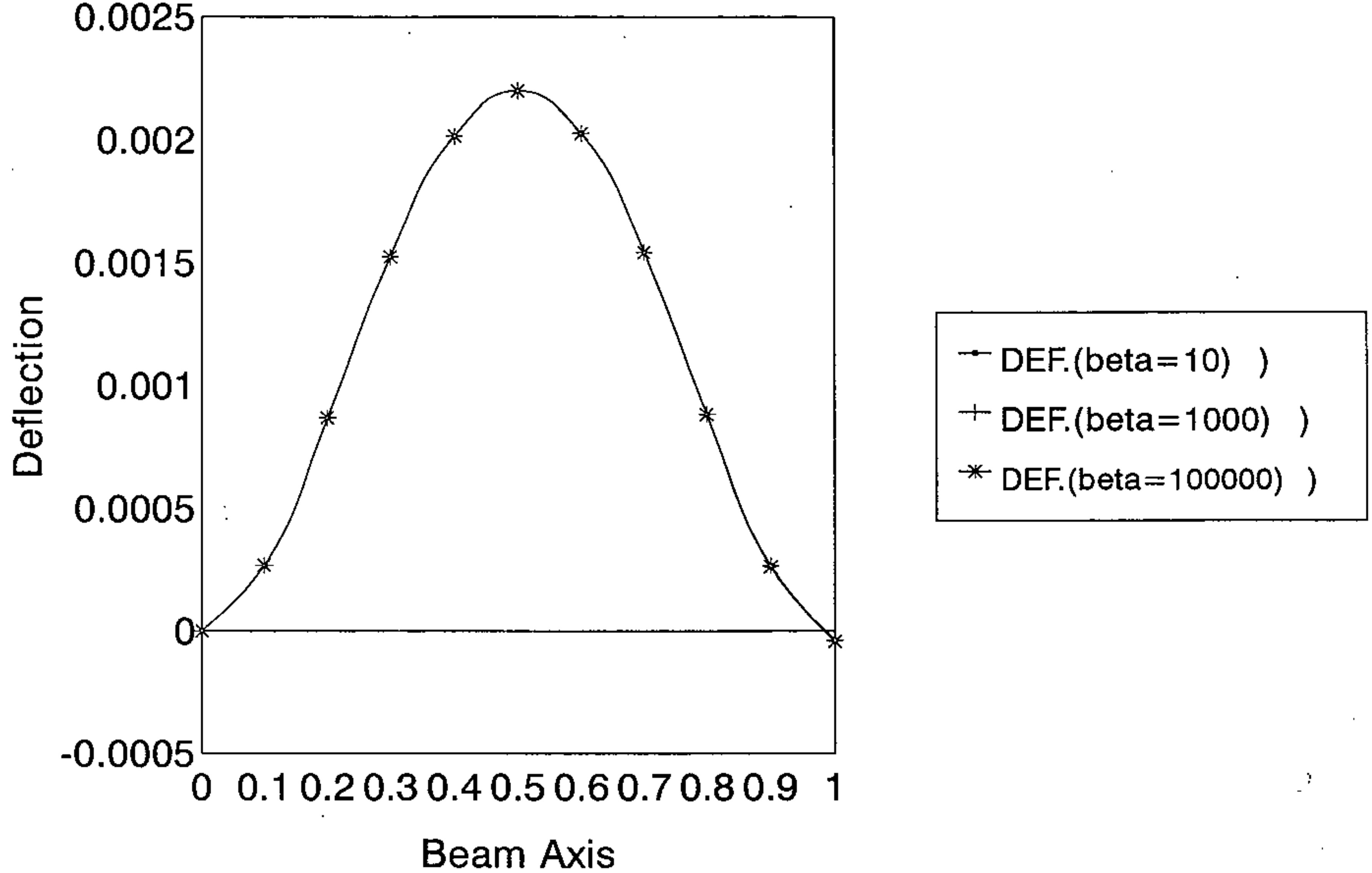


Fig.(5-19a)Deflection versus beam axis for clamped beam/ sinusoidal load with alfa=10 ,beta variable

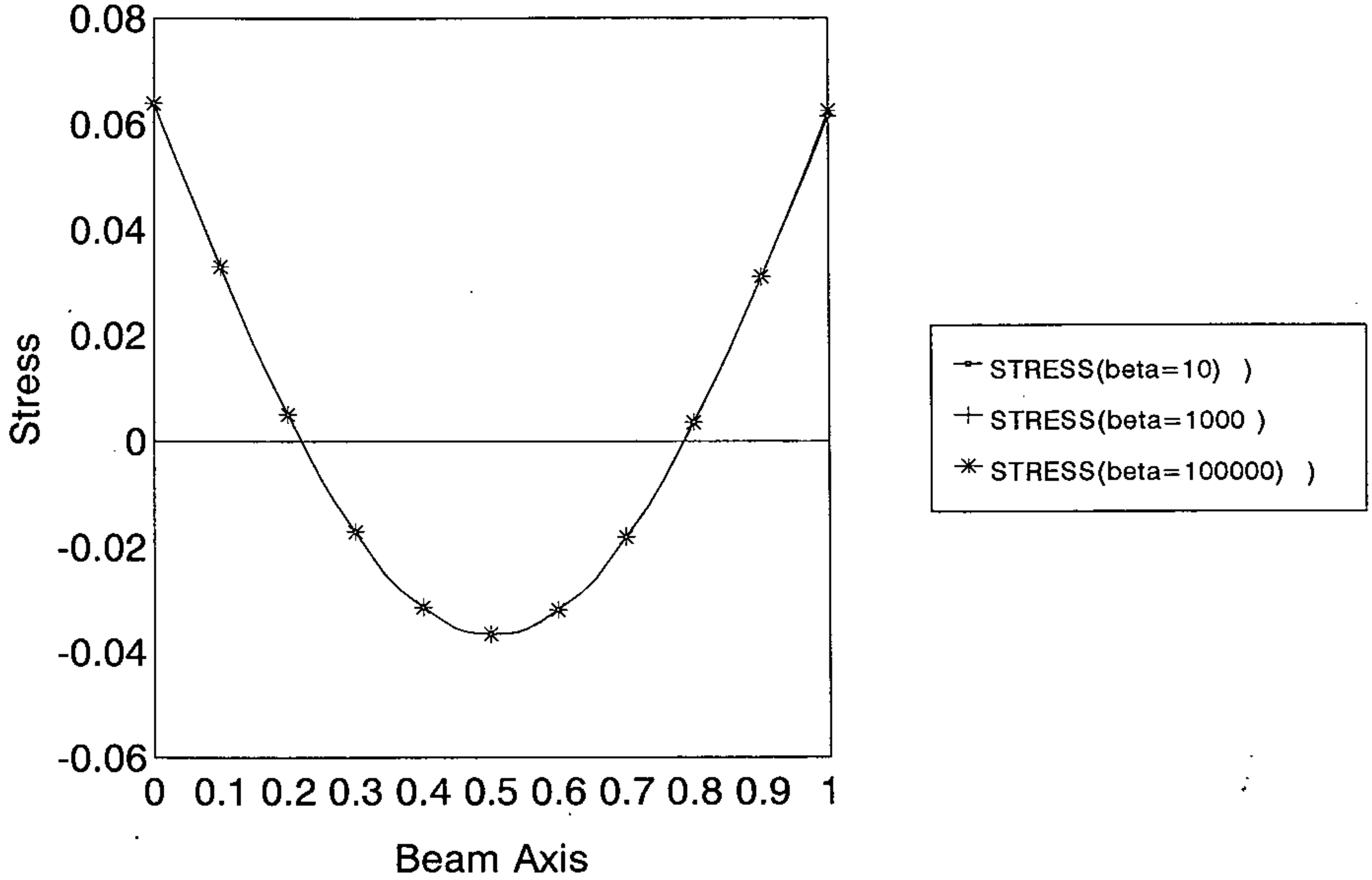


Fig.(5-19b)Stress versus beam axis for clamped beam/ sinusoidal load with alfa=10 ,beta variable

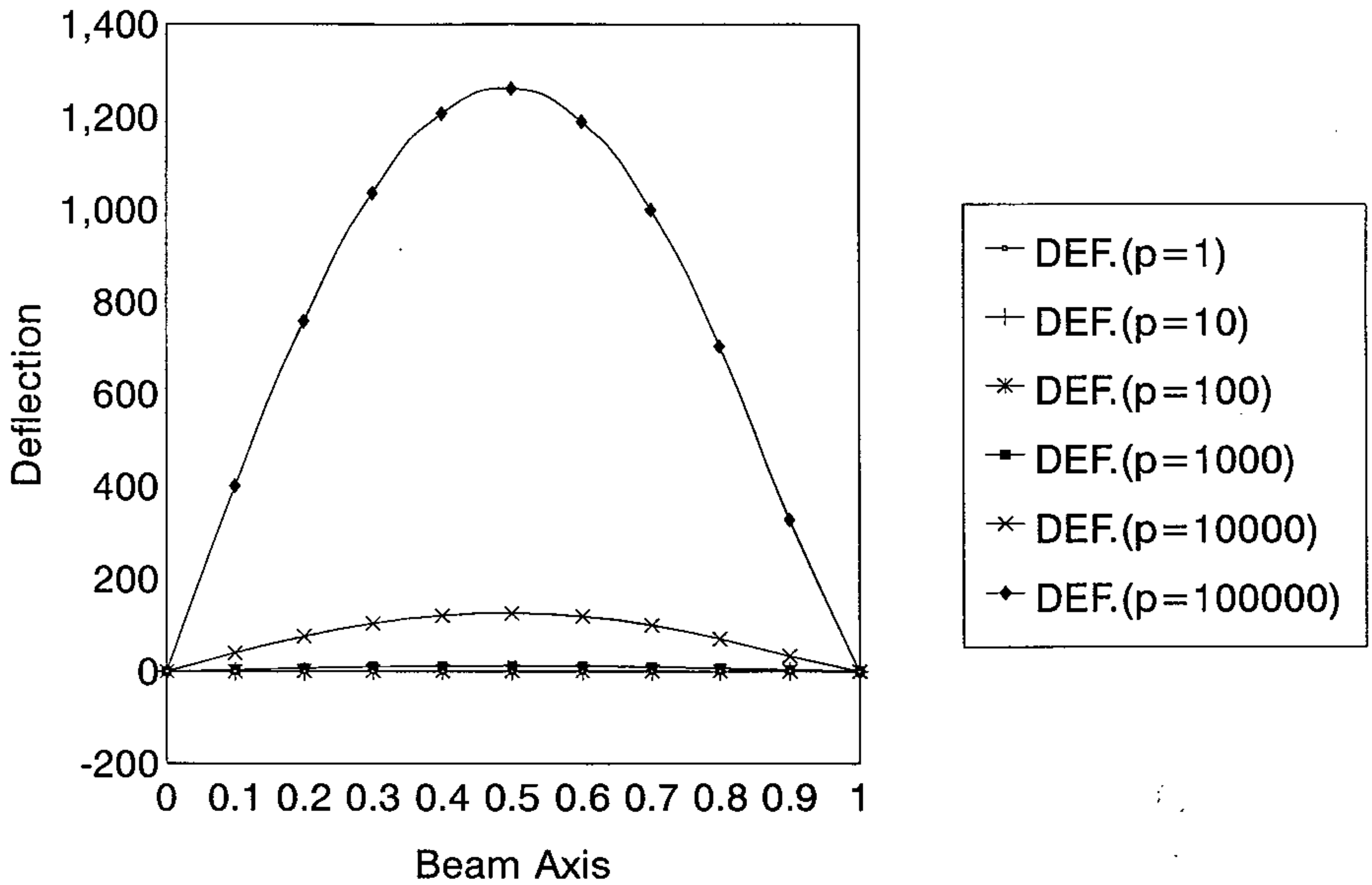


Fig.(5-20a) Deflection versus beam axis for simply supported beam /constant load with load variable, alfa=0 ,beta=0

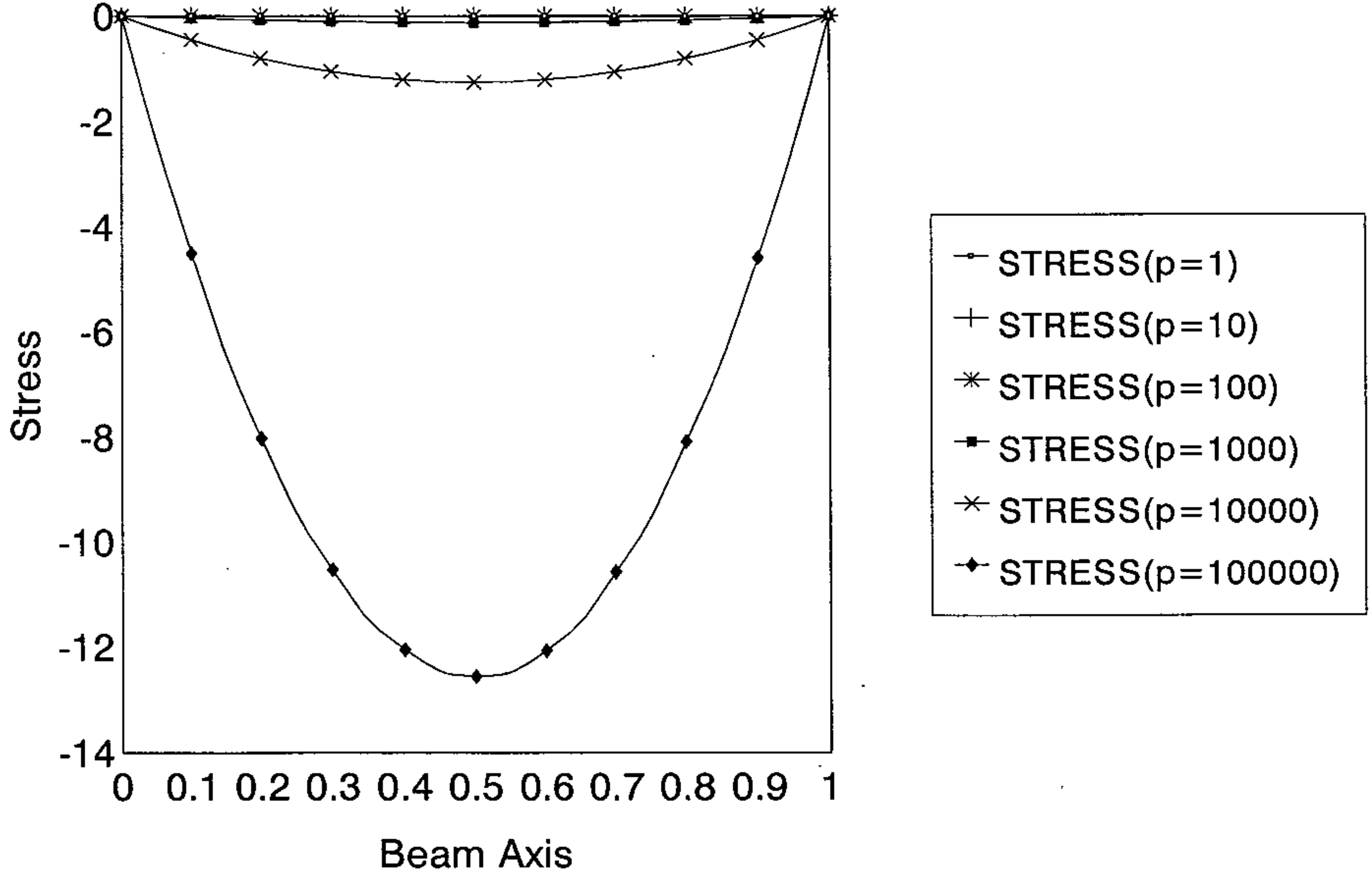


Fig.(5-20b) Stress versus beam axis for simply supported beam/constant load with load variable, alfa=0 ,beta=0

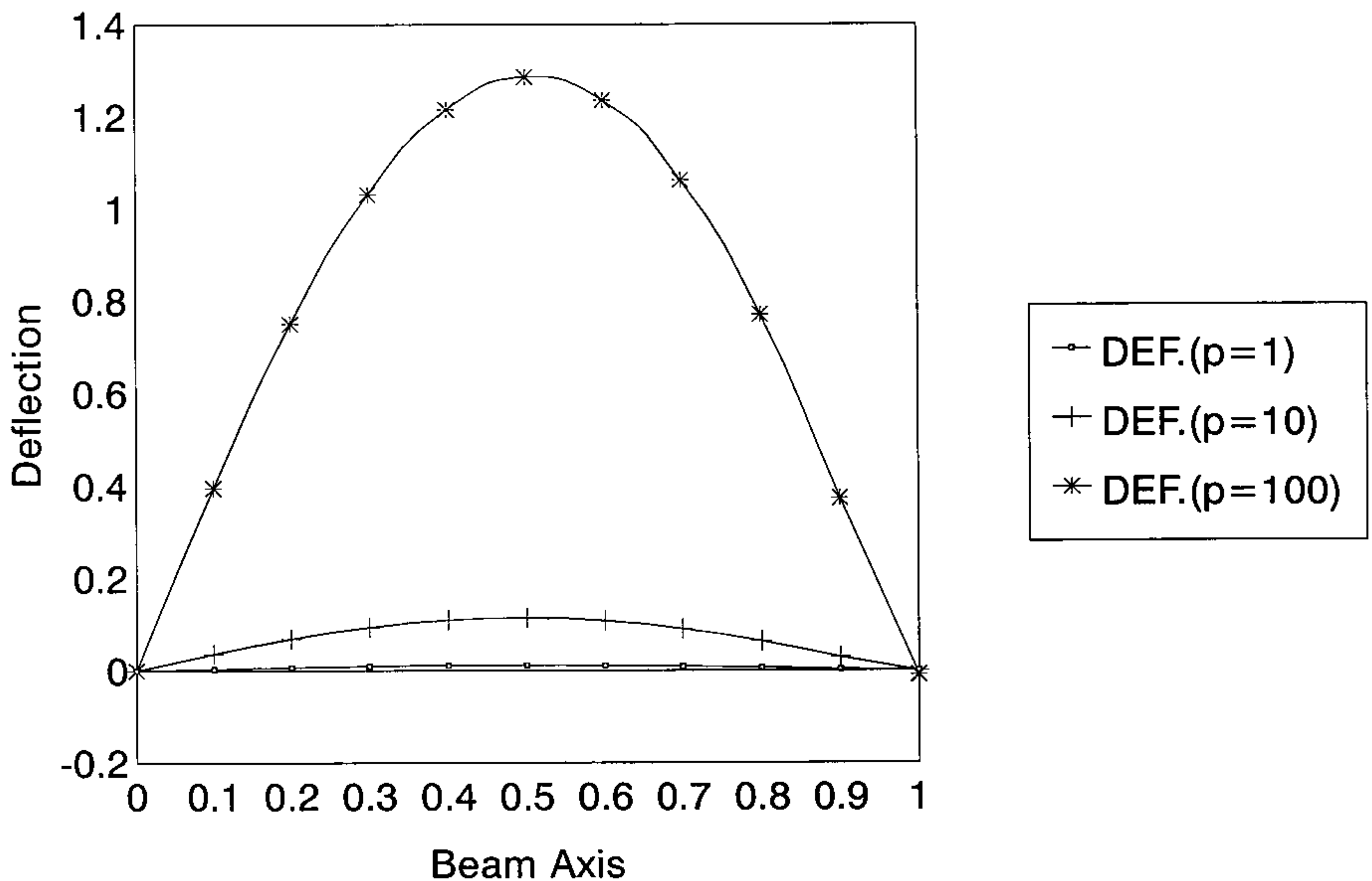


Fig.(5-21a) Deflection versus beam axis for simply supported beam/constant load with load variable, alfa=10 ,beta=10

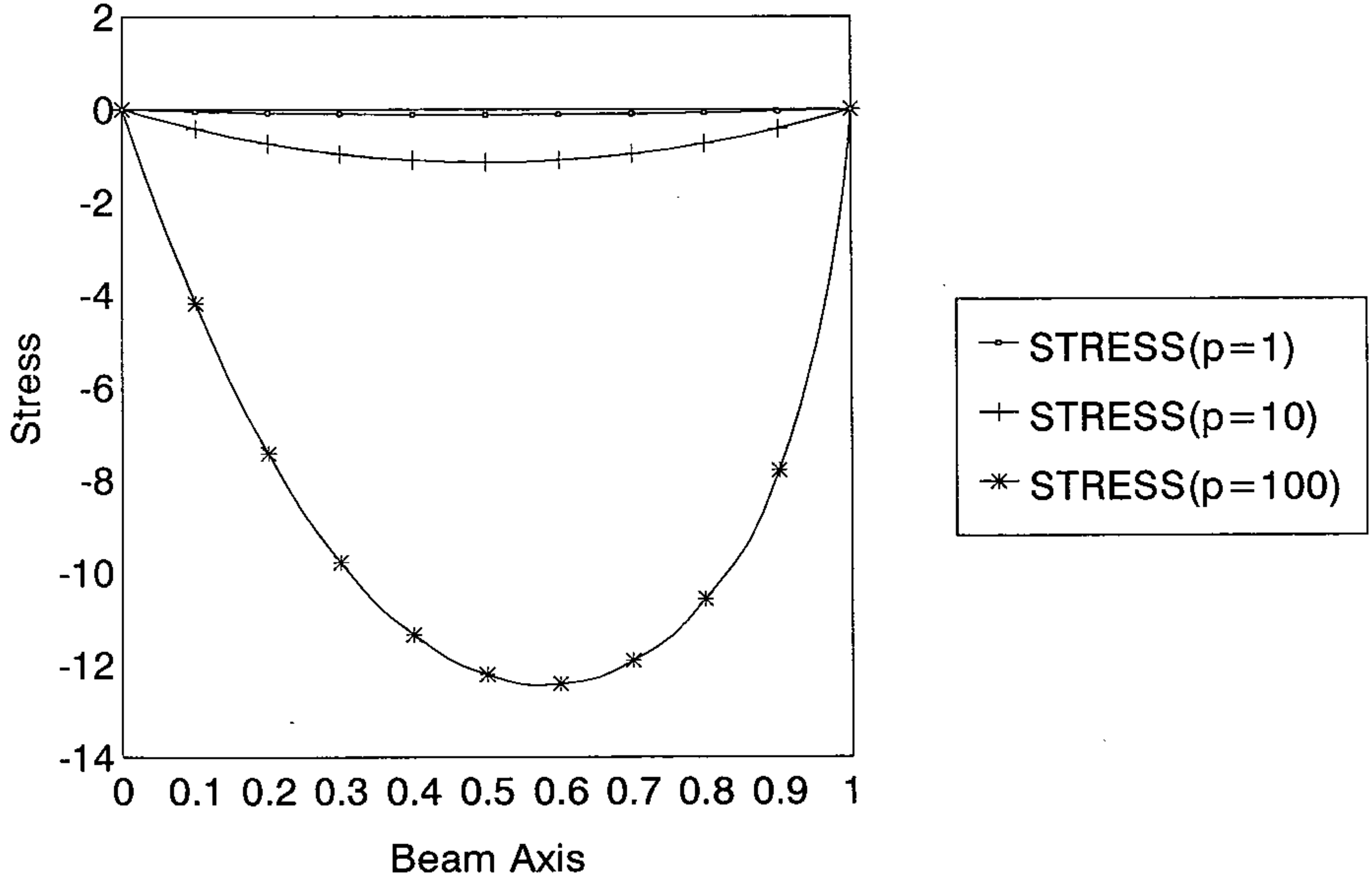


Fig.(5-21b) Stress versus beam axis for simply supported beam/constant load with load variable, alfa=10, beta=10

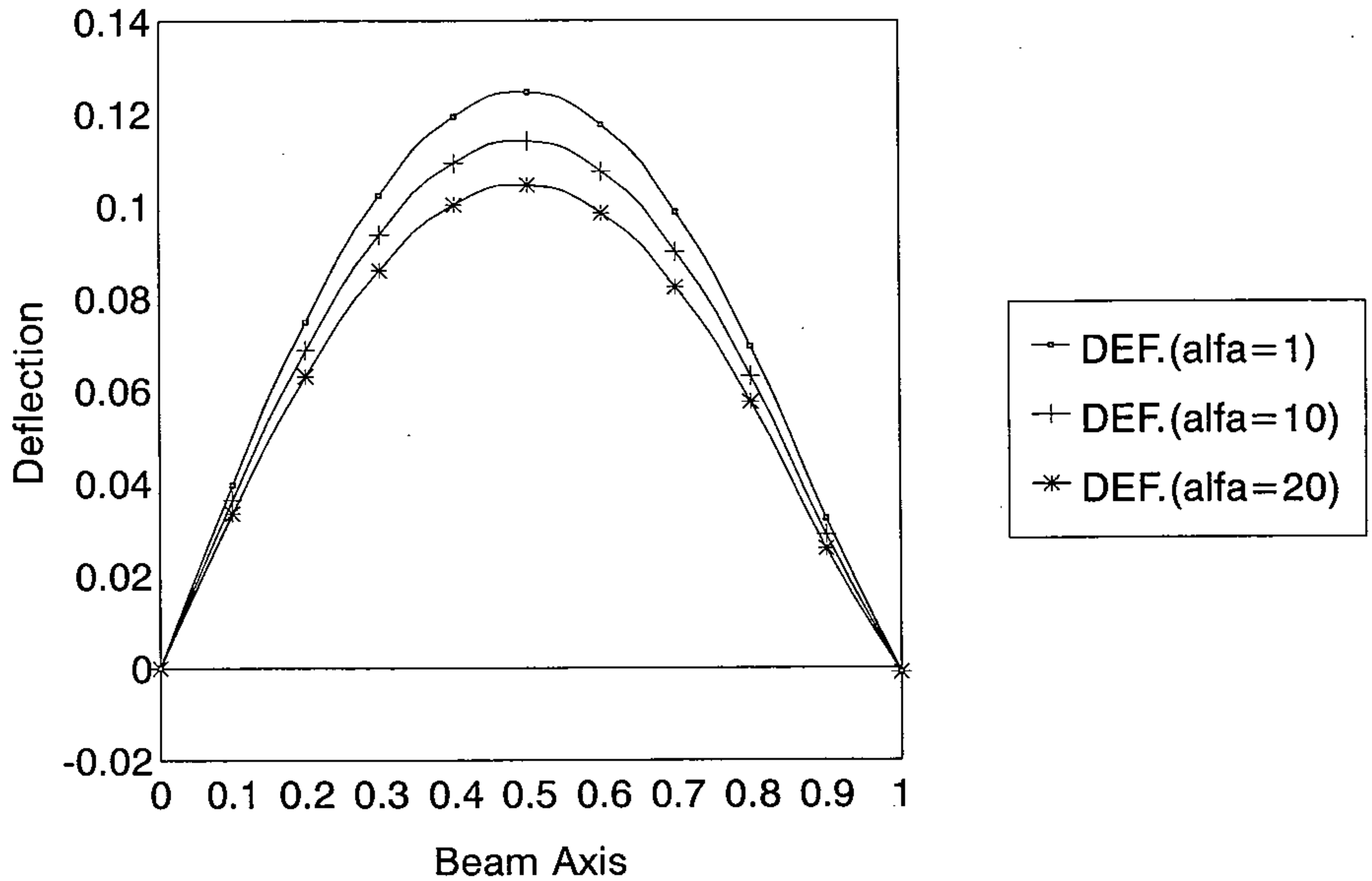


Fig.(5-22a) Deflection versus beam axis for simply supported beam/constant load with load=10, alfa variable, beta=10

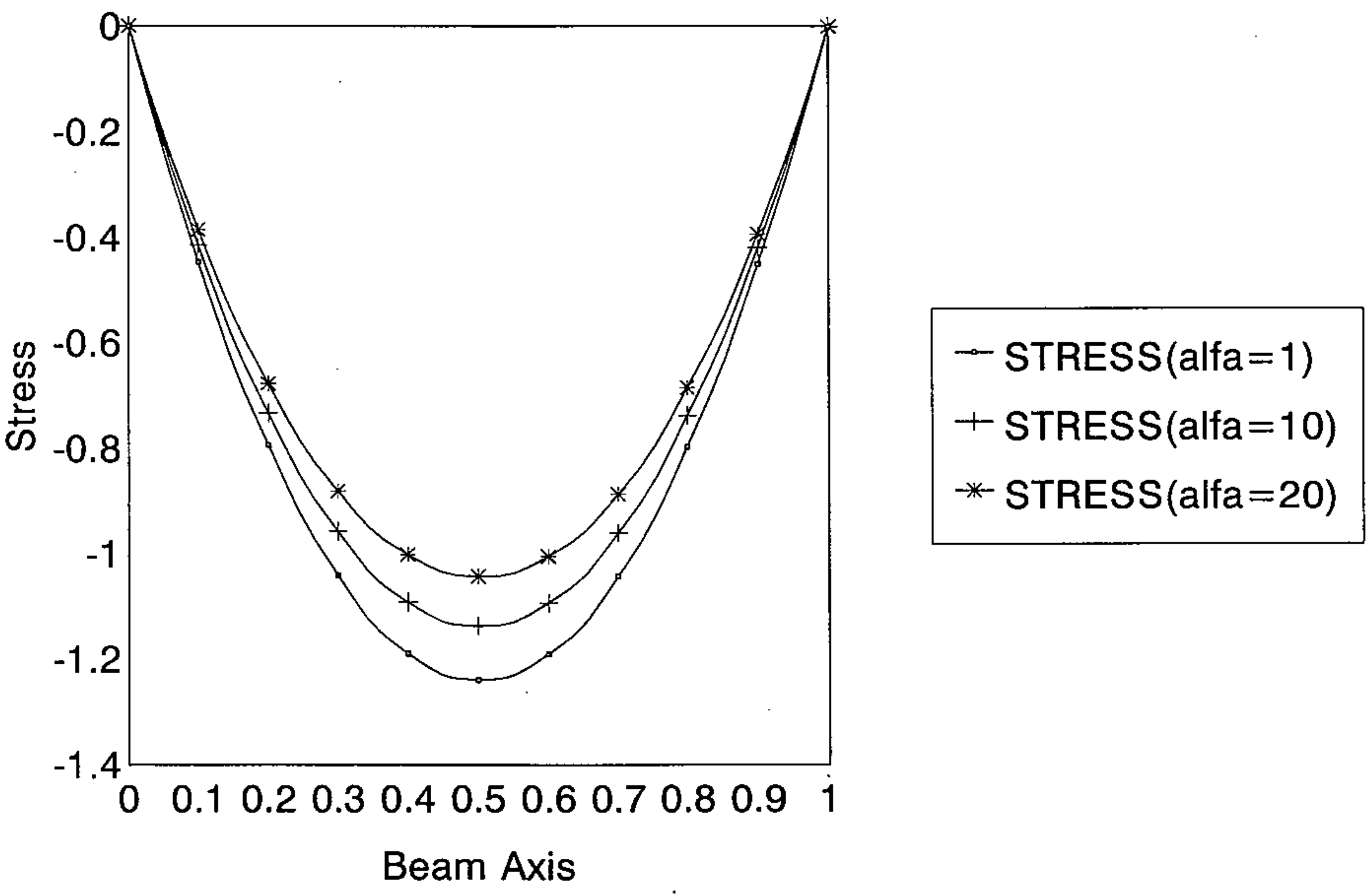


Fig.(5-22b) Stress versus beam axis for simply supported beam/constant load with load=10, alfa variable, beta=10

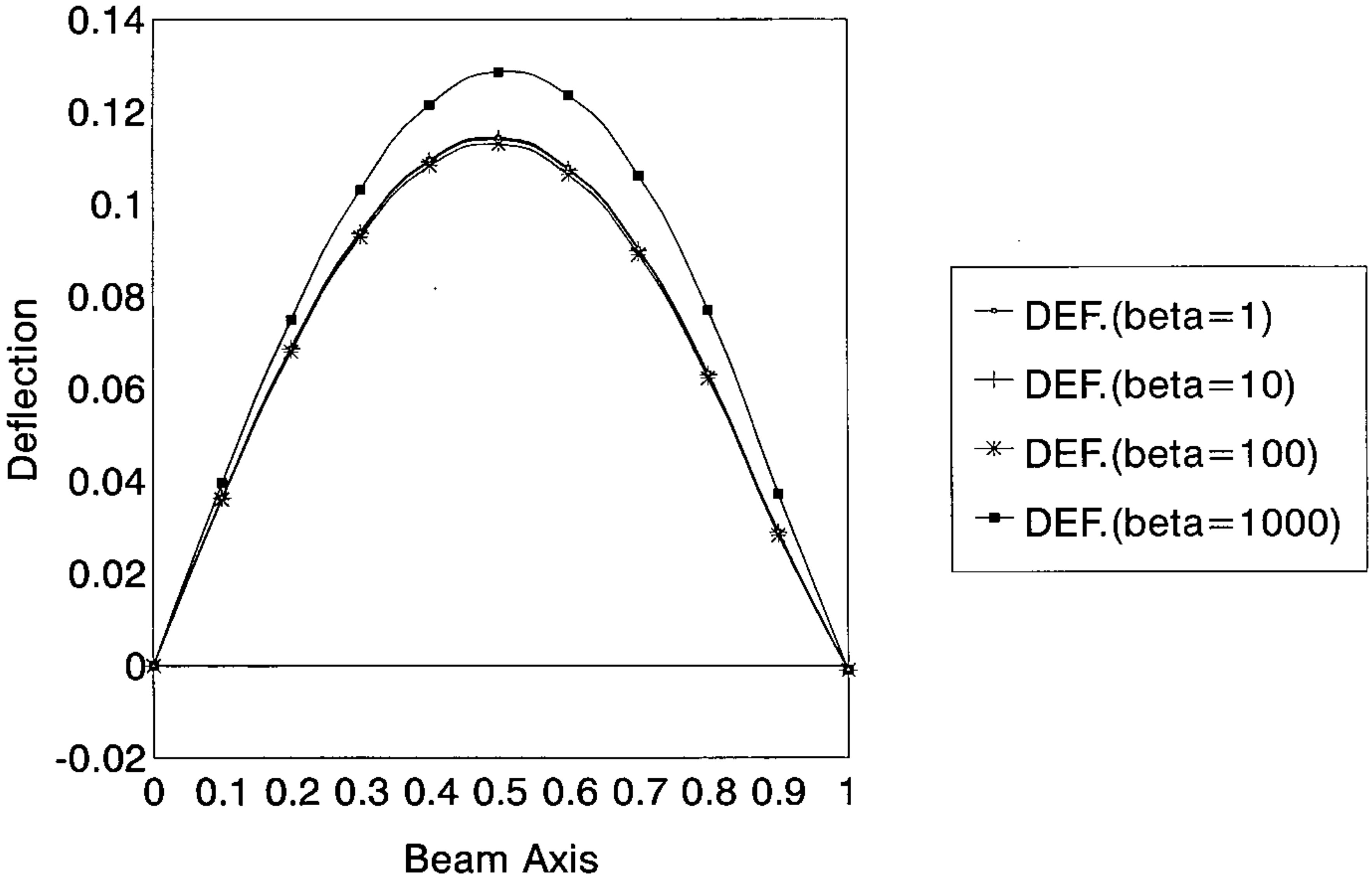


Fig.(5-23a) Deflection versus beam axis for simply supported beam/constant load with load=10, alfa=10 ,beta variable

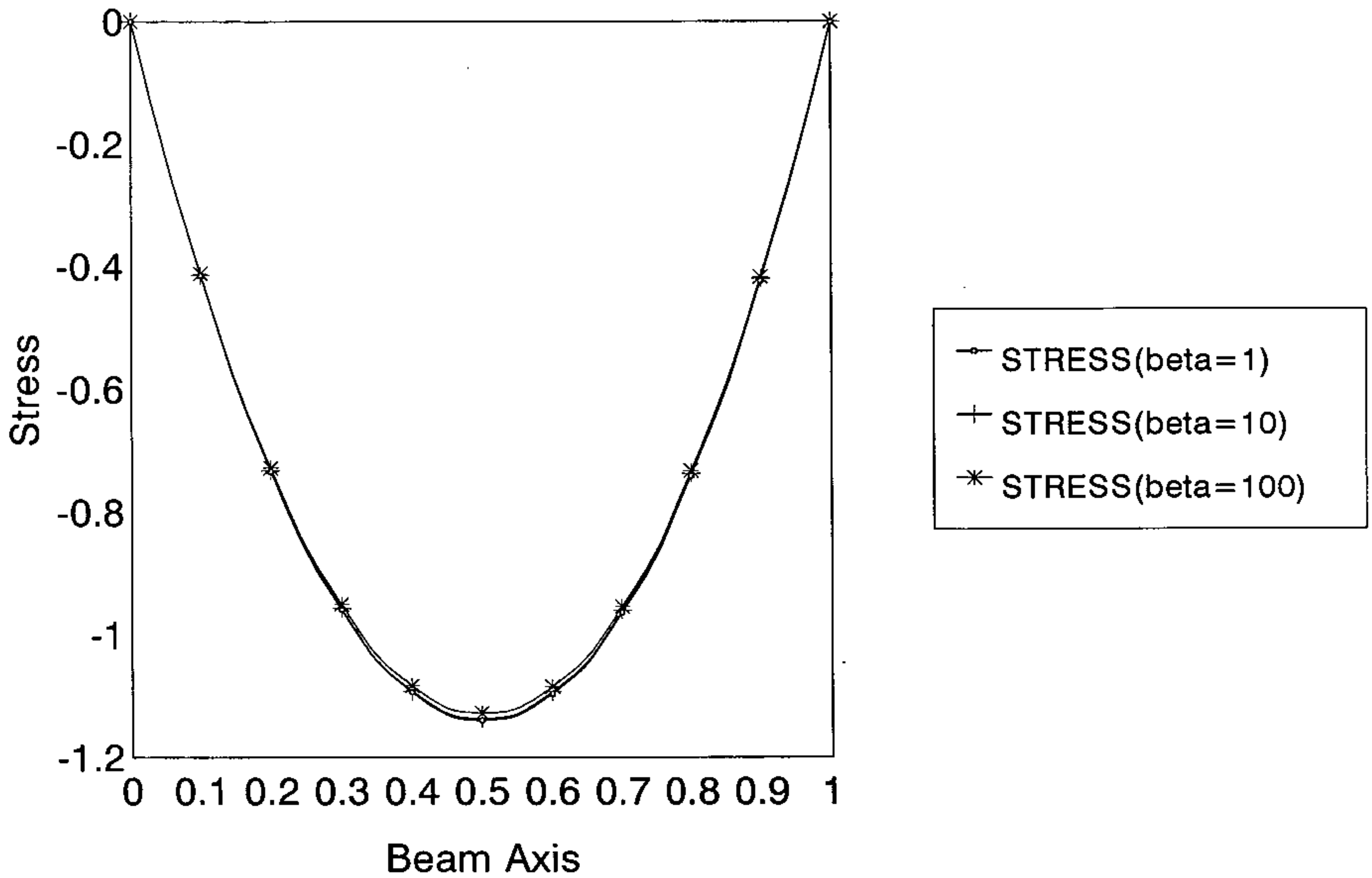


Fig.(5-23b) Stress versus beam axis for simply supported beam/constant load with load=10, alfa=10 ,beta variable

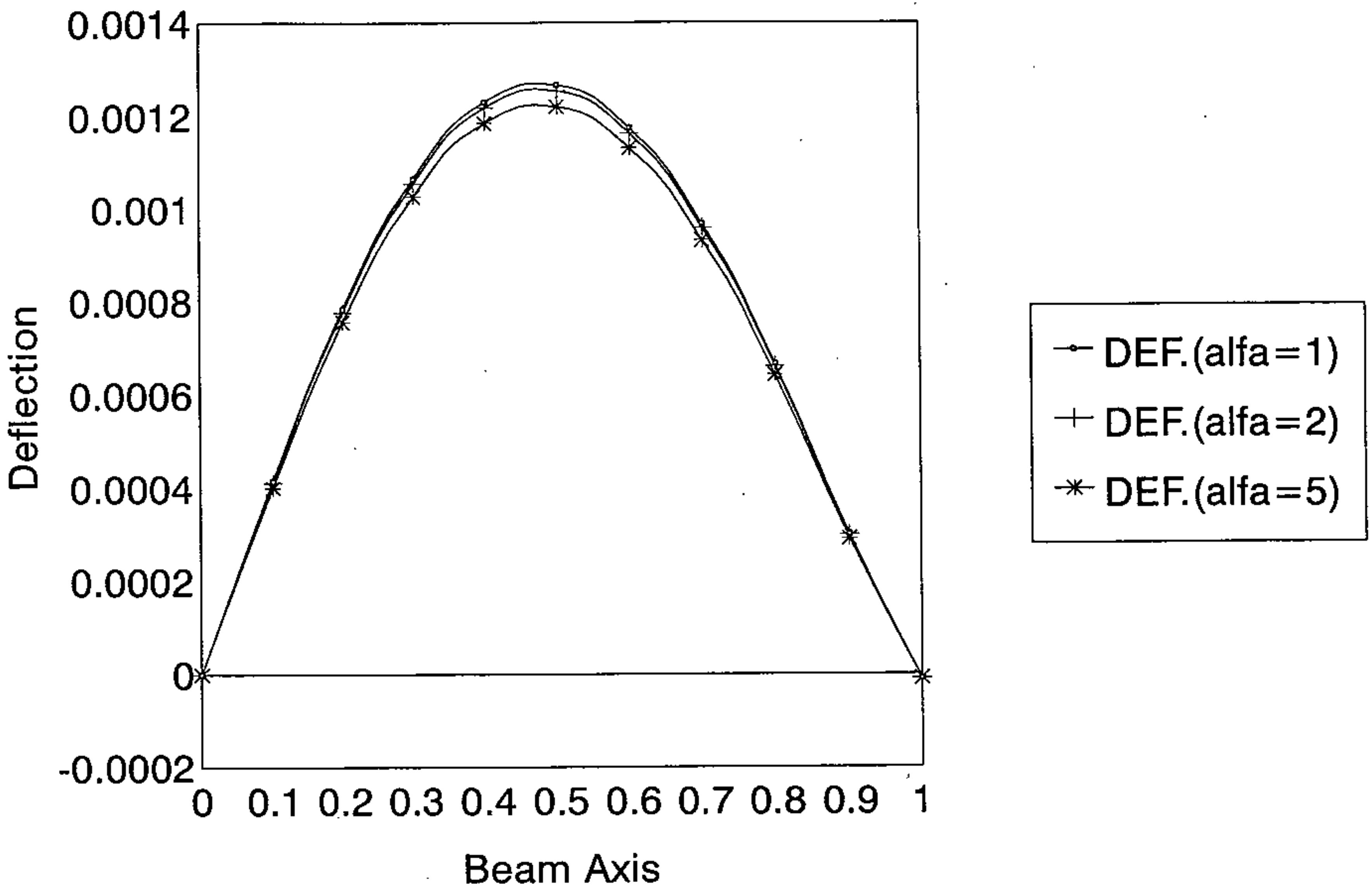


Fig.(5-24a) Deflection versus beam axis for simply supported beam/cubic load with alfa variable , beta= 10

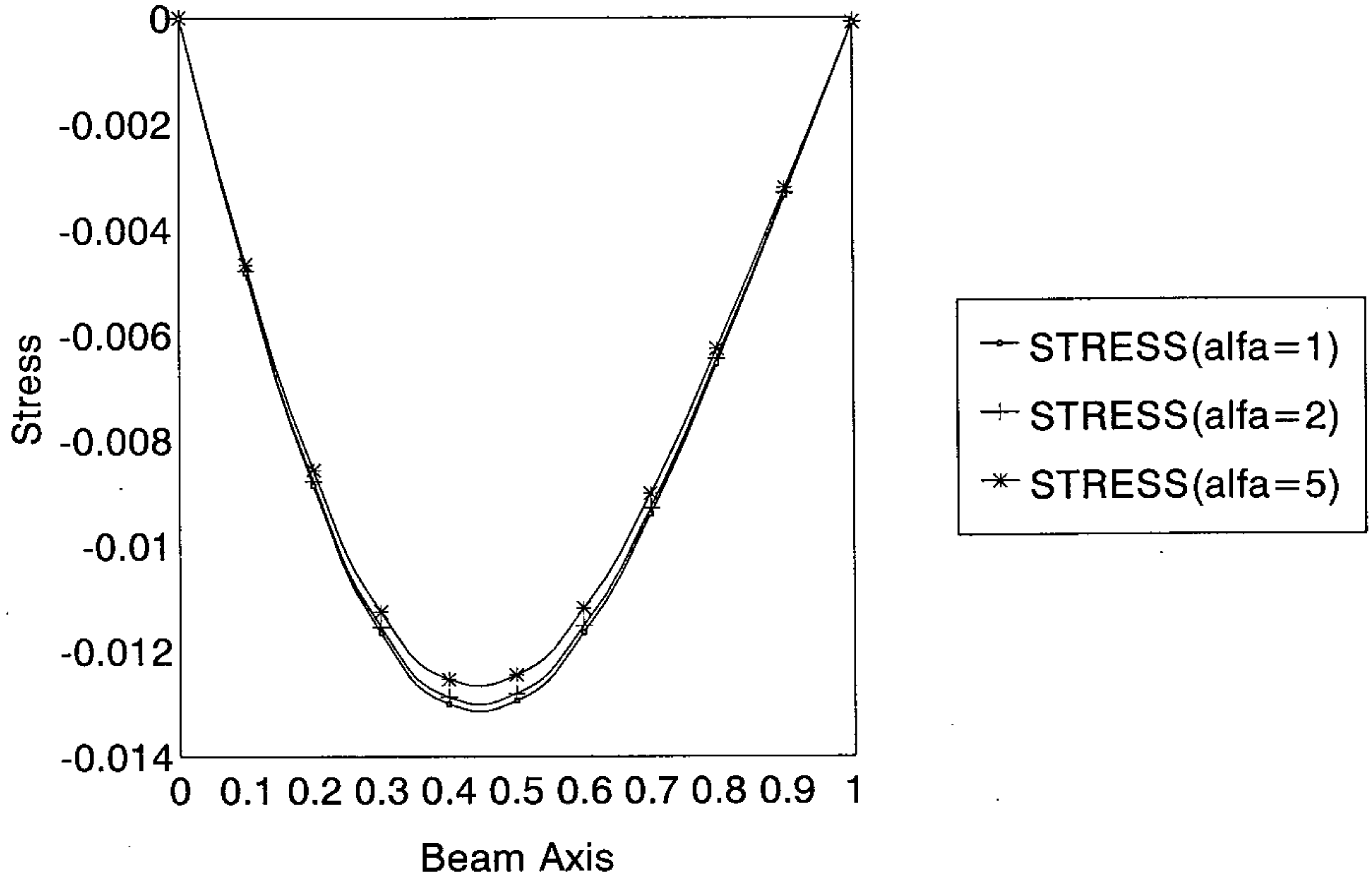


Fig.(5-24b) Stress versus beam axis for simply supported beam/cubic load with alfa variable , beta=10

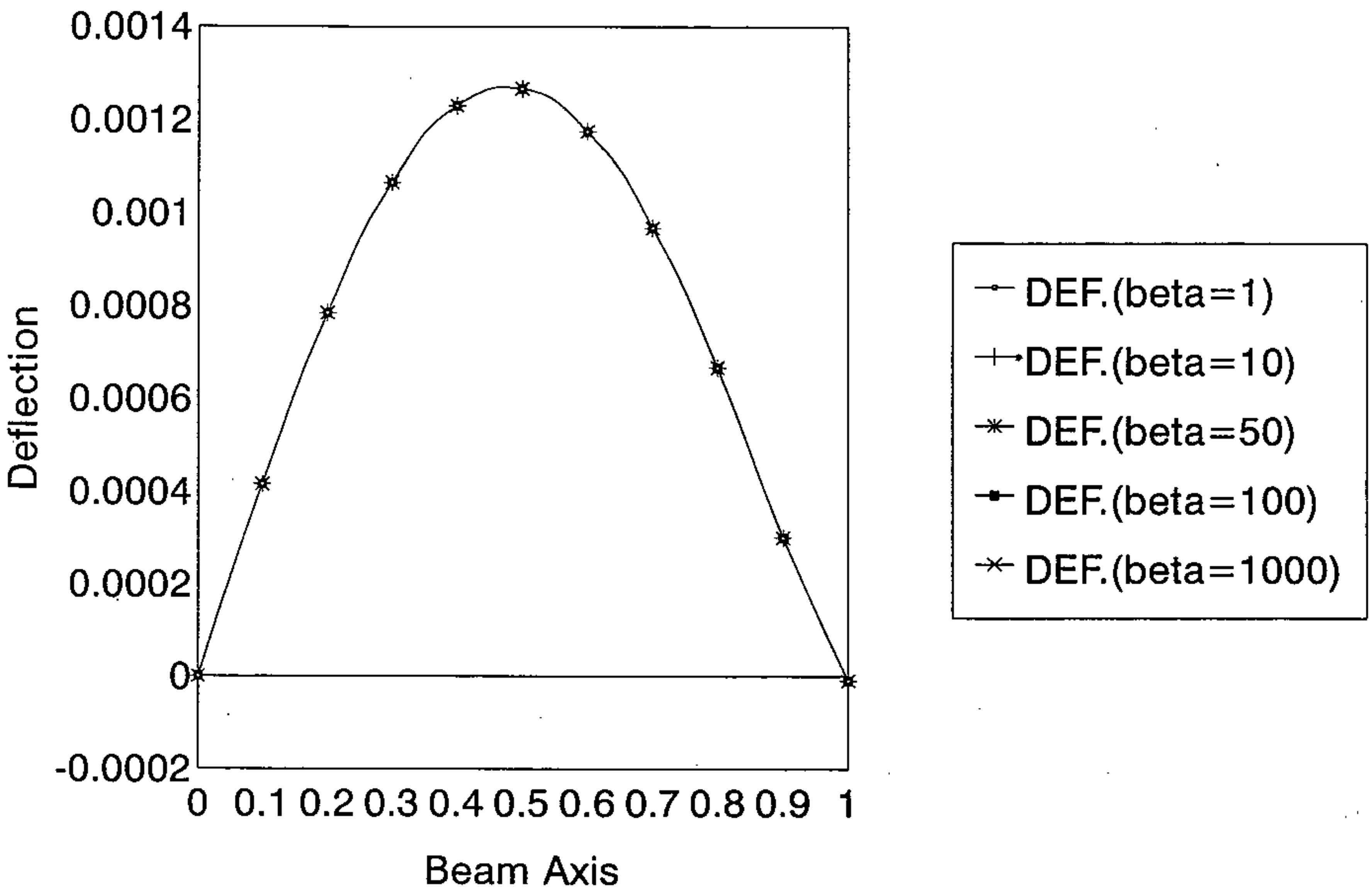


Fig.(5-25a) Deflection versus beam axis for simply supported beam/cubic load with alfa=1 , beta variable

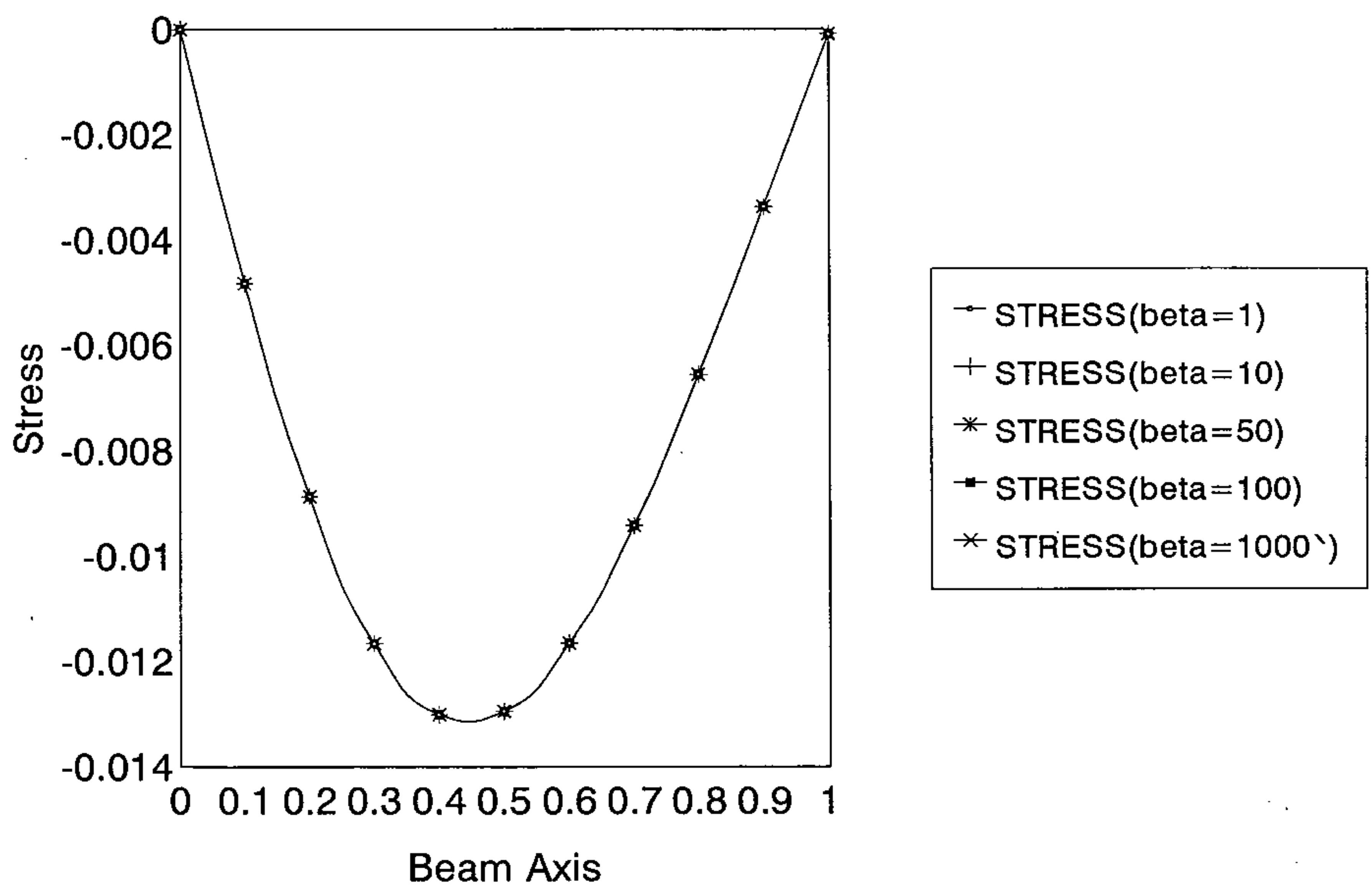


Fig.(5-25b) Stress versus beam axis for simply supported beam/cubic load with alfa=1 , beta variable

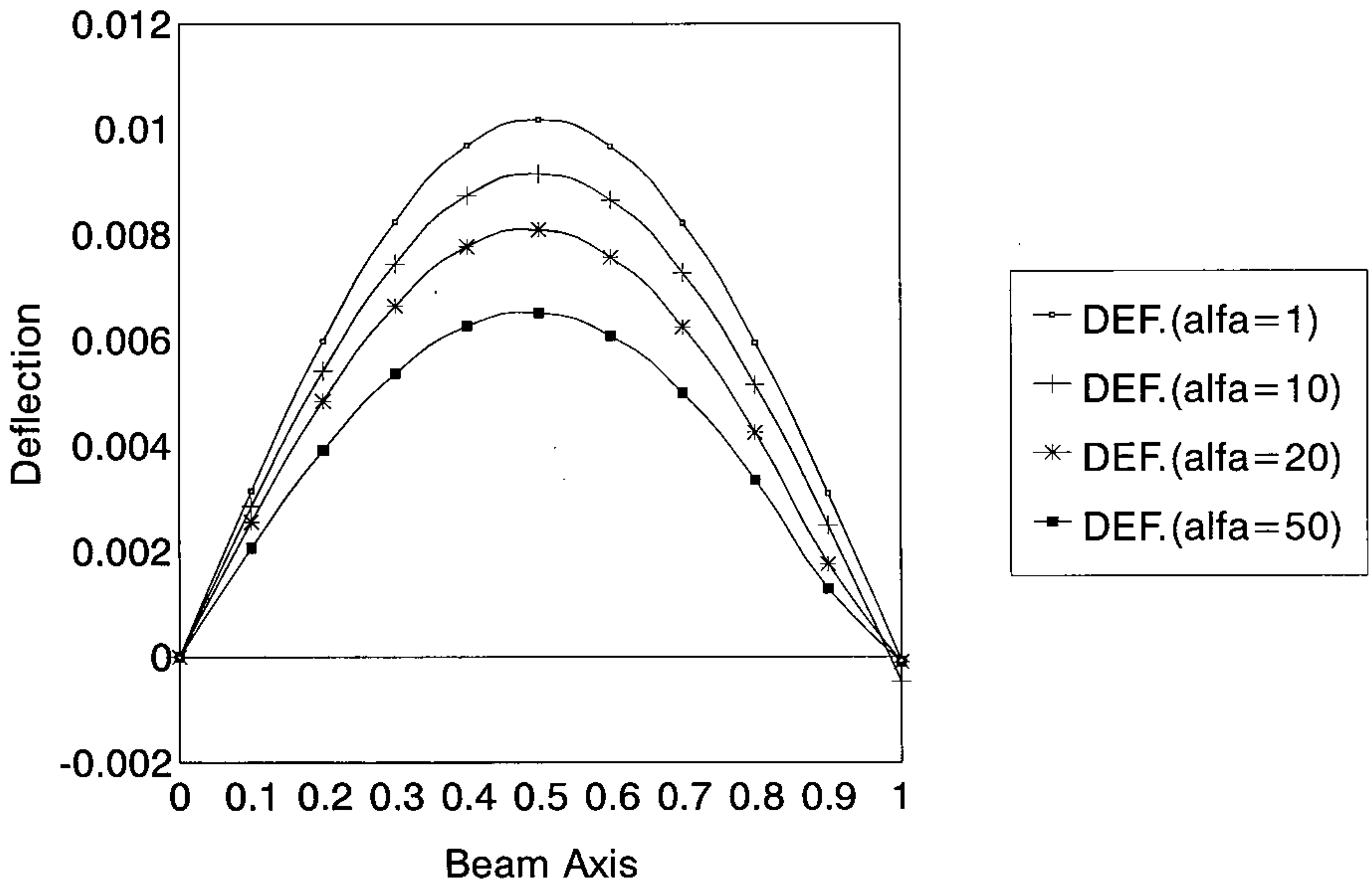


Fig.(5-26a) Deflection versus beam axis for simply supported beam/sinusoidal load with alfa variable ,beta=10

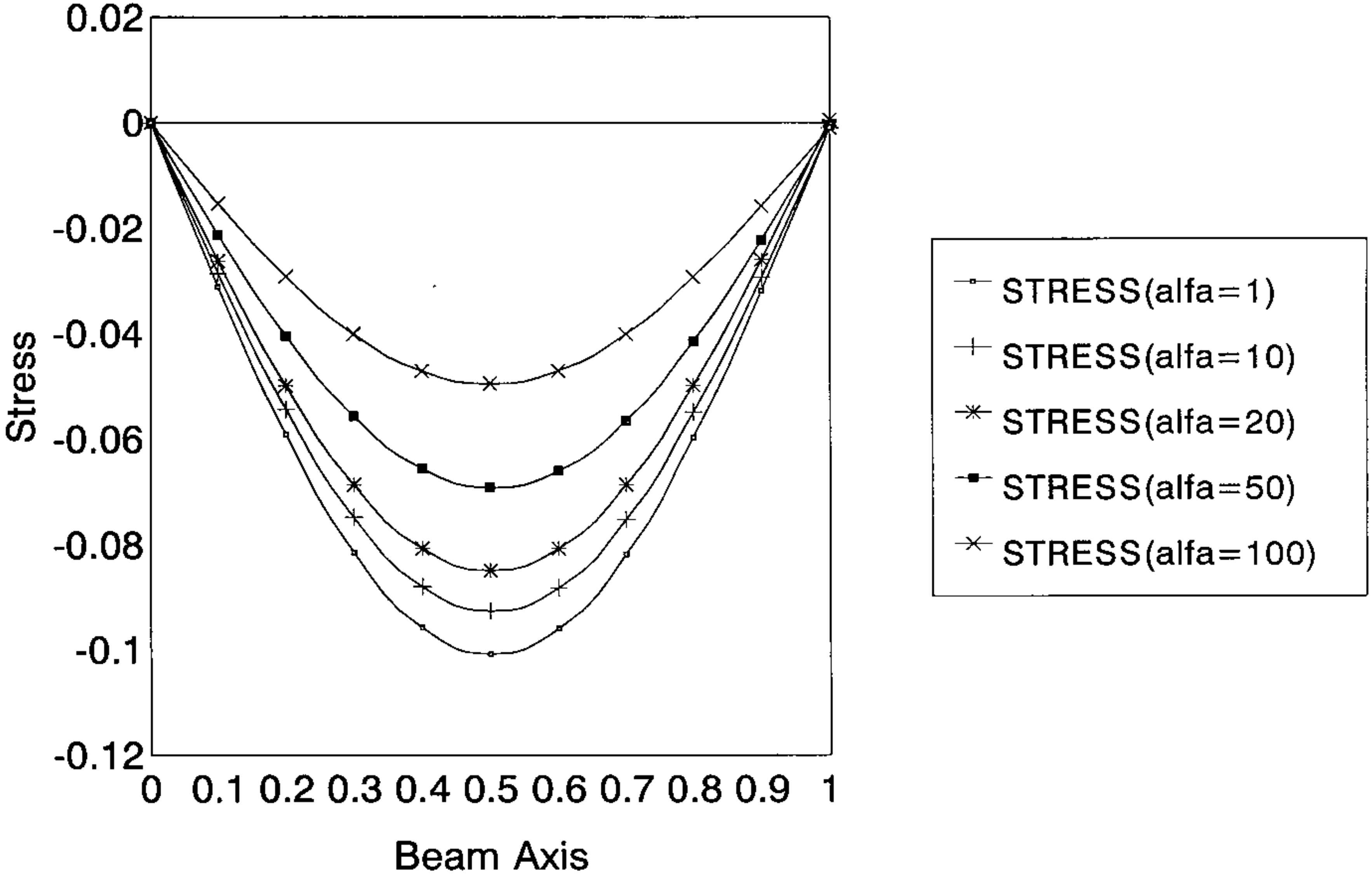


Fig.(5-26b) Stress versus beam axis for simply supported beam/sinusoidal load with alfa variable, beta=10

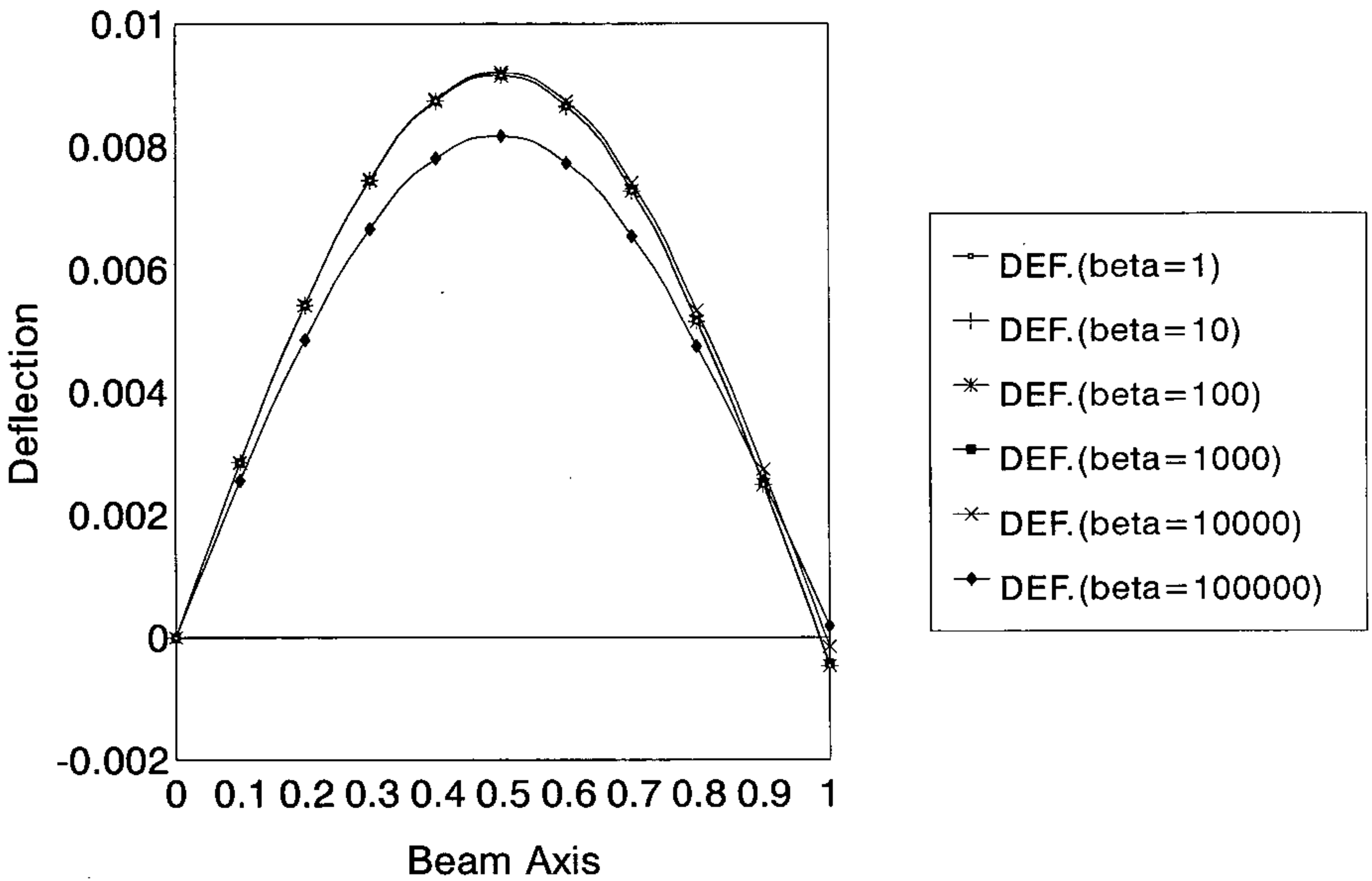


Fig.(5-27a) Deflection versus beam axis for simply supported beam/sinusoidal load with alfa=10, beta variable

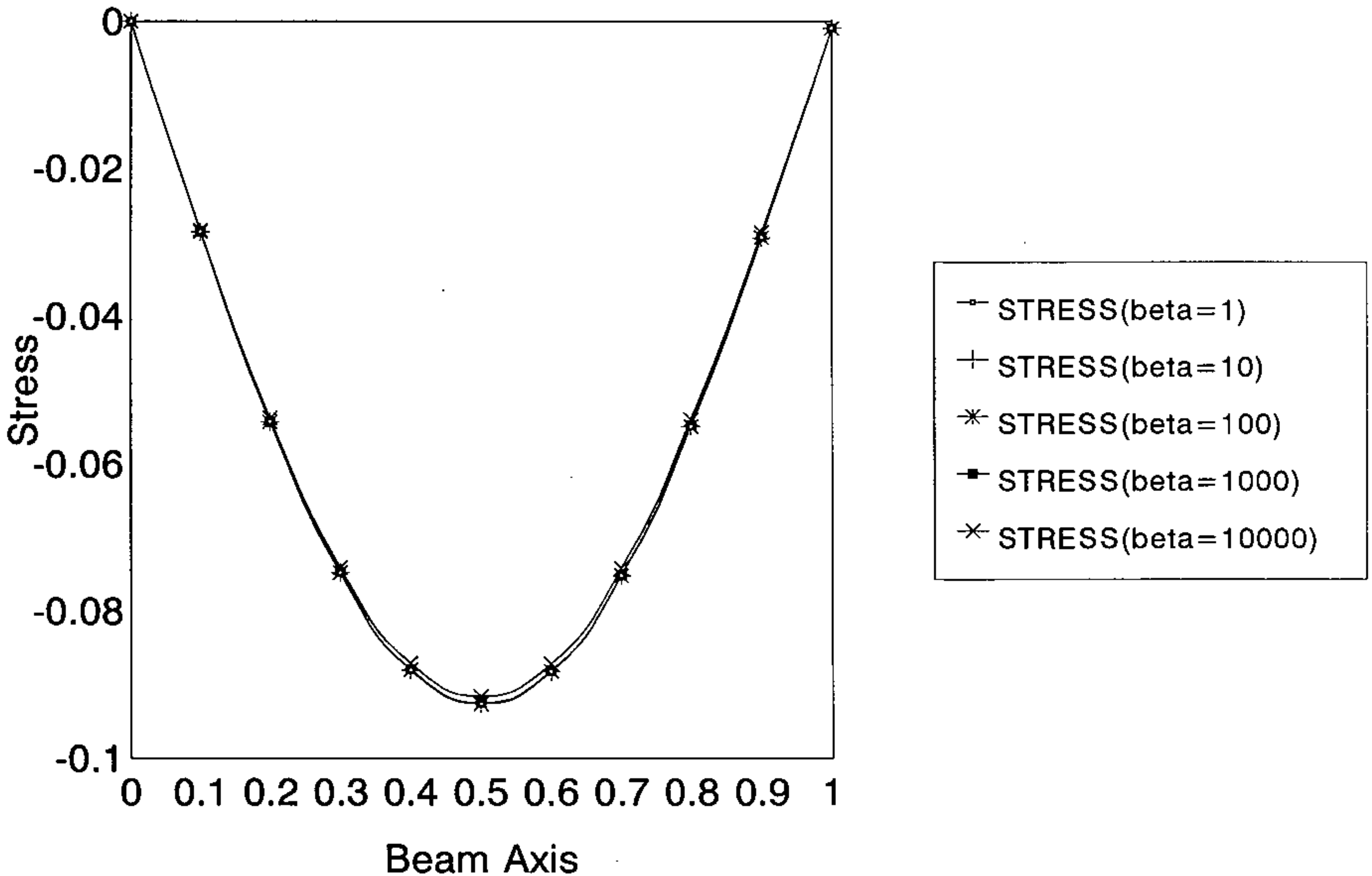


Fig.(5-27b) Stress versus beam axis for simply supported beam/sinusoidal load with alfa=10, beta variable

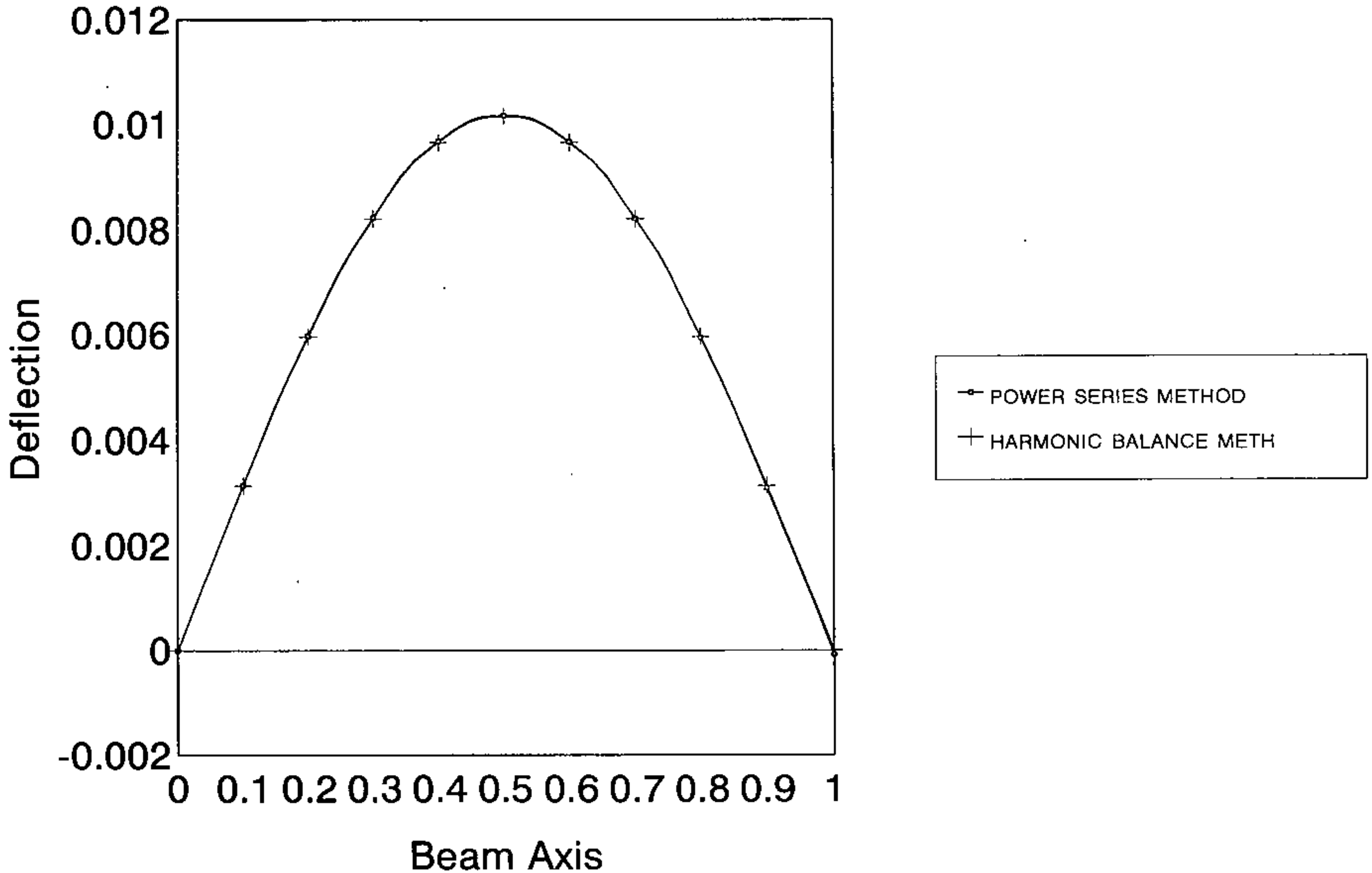


Fig.(5-28a) Harmonic Balance Method & Power Series Method comparison at alfa=1.0 , beta=1.0

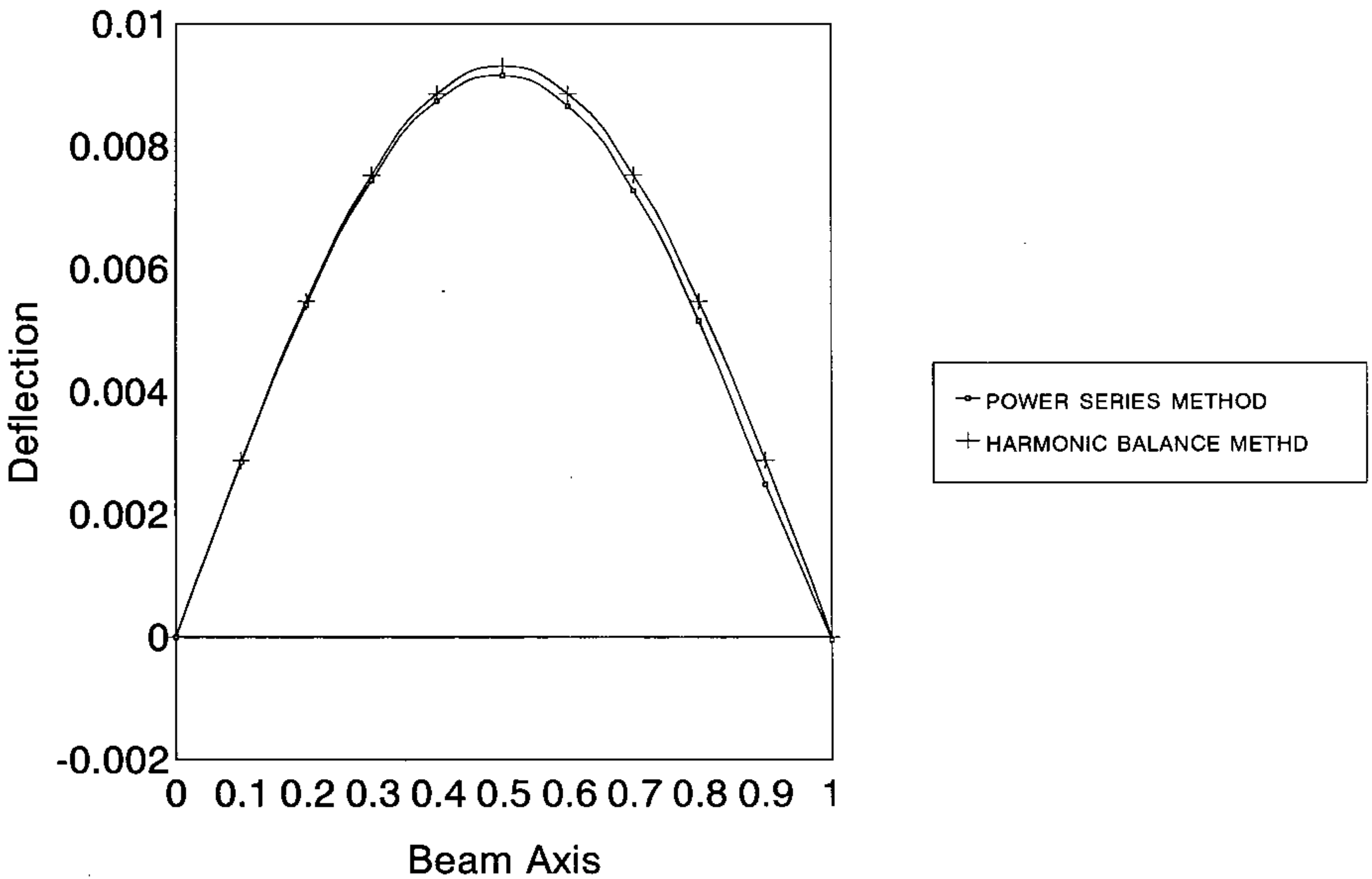


Fig.(5-28b) Harmonic Balance Method & Power Series Method comparison at alfa= 10 , beta=10

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

6-1 Conclusions

Several important points have emerged from this work, which can be summarized :

1- The power series method is a very powerful method, it can be programmed easily, double precision results of the method minimized the truncation and round off errors due to large mathematical operations since the problem is non linear.

2-Harmonic balance method has an exact solution for simply supported beam sinusoidal load , Its results have the same trend of the power series solution for the same case which yields the effectiveness of power series method .

3- The running time of the power series program was mainly dependent on the polynomial order N, it was found from convergence study of the power series solution for deflections and maximum bending stresses that the 20^{th} polynomial order is the most suitable when for establishing a solution to the problem.

4- The deflection of the beam increases as the load increases, and it is maximum value for symmetric loads at x=0.5 which is expected while it remains zero at ends for clamped and simply supported beams, for unsymmetric loads the deflection and stress depend on the shape of load .

5- Maximum bending stress has positive and negative values over the beam and it increases as the load increases.

6- The effect of linear stiffness of foundation α at constant values of the parameters, (loads and β) makes the beam more hardened as it increases causes less deflection of the beam i.e. more load is needed to deflect the beam.

7- The effect of non linear stiffness of foundation β has the same trend of α but in very small effect in comparison with the effect of α.

8- The linear and non linear stiffness (α & β) have a great effect on the solution of the fourth order differential equation in comparison when they have zero values .

9- The tolerance at boundary conditions of nonlinear problem has a great effect on determining the coefficients of power series polynomial .

10- There are limits of α & β values that no power series solution can be found above these limits those limits depend on the study case parameters as type of beam, kind of loads and boundary conditions.

6-2 Recommendations

The following is recommended to be investigated on next researches :

1- Study other types of beams by power series method as free end beams (cantilevers) deflections and stresses.

2- Study one point load on beams by power series method .

3- Investigate the effect of the cross section of the beams by power series method .

4- Compare if possible other exact or approximate methods by power series method .

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APPENDICES

Appendix A : Power Series .

The power series is on infinite series in power of ($x-x_0$) of the form

$$\sum_{m=0}^{\infty} a_m (x-x_0)^m = a_0 + a_1(x-x_0) + a_2(x-x_0)^2 + \dots \qquad \text{(A-1)}$$

when a_0, a_1, a_2 ... are constants, called the coefficients of series, and x is a variable,

if in particular $x_0=0$

we obtain a power series in power of x

$$\sum_{m=0}^{\infty} a_m x^m = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots \qquad \text{(A-2)}$$

we assume that all variable and constants are real

So the familiar examples of power series are

$$\frac{1}{1-x} = \sum_{m=0}^{\infty} x^m = 1 + x + x^2 + \dots \qquad |x| < 1 \qquad \text{(A-3)}$$

$$e^x = \sum_{m=0}^{\infty} \frac{x^m}{m!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \qquad \text{(A-4)}$$

if we have differential equation as

$$y'' + P(x)y' + q(x)y = 0 \qquad \text{(A-5)}$$

we should represent P(x) & q(x) by power series in power of x, often P(x) & q(x) are polynomials.

we assume the solution in form of a power series with unknown coefficients.

$$y = \sum_{m=0}^{\infty} a_m x^m = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ... \qquad \text{(A-6)}$$

$$y' = \sum_{m=1}^{\infty} m a_m x^{m-1} = a_1 + 2a_2 x + 3a_3 x^2 + ... \qquad \text{(A-7)}$$

$$y'' = \sum_{m=2}^{\infty} m(m-1) a_m x^{m-2} = 2a_2 + 3\times 2a_3 x + 4\times 3a_4 x^3 + ... \qquad \text{(A-8)}$$

insert all these series in differential equation, then we collect like power of x and equate sum of the coefficients of each occurring power x to zero, starting with constant terms, the terms containing x, the terms containing x^2 etc.

This gives relations from which we can determine the unknowns coefficients in successively

Example : solve $y'' + y = 0$ (A-9)

Solution : insert equation. (A-8) & (A-6), we obtain

$$(2a_2 + 3\times 2a_3\, x + 4\times 3a_4\, x^2 + ...) +$$

$$(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ...) = 0 \qquad \text{(A-10)}$$

collecting like powers of x, we find:

$$(2a_2 + a_0) + (3\times 2a_3 + a_1) + (4\times 3a_4 + a_2)\, x^2 = 0 \qquad \text{(A-11)}$$

Equating coefficients of each power of x to zero

$2a_2 + a_0 = 0$ coeff. of x^0

$3\times 2a_3 + a_1 = 0$ coeff. of x^1

$4\times 3a_4 + a_2 = 0$ coeff. of x^2 (A-12)

Solving these equations, we see that a_2, a_4, a_6 , ... may be expressed in terms of a_0 and a_3 , a_5 , a_7 , ... may be expressed in term of a_1 :

$a_2 = \frac{-a_0}{2!}$, $a_3 = -\frac{a_1}{3!}$, $a_4 = \frac{a_2}{4\times 3} = \frac{a_0}{4!}$ and so on

a_0 and a_1 are arbitrary

$$y = a_0 + a_2 x - \frac{a_0}{2!}x^2 - \frac{a_1}{3!}x^3 + \frac{a_0}{4!}x^4 + \ldots \qquad \text{(A-13)}$$

$$y = a_0\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots.\right) + a_1\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}+\ldots.\right)$$

$$= a_0 \cos x + a_1 \sin x \qquad \text{(A-14)}$$

Convergence of power series :

if the sequence of partial sums converges to finite limit L ; we write

$\sum_{m=0}^{\infty} a_m = L$ then the sum converges to L , we call L the sum of series.

if partial sums diverges then

the sum $\sum_{m=0}^{\infty} a_m$ diverges ,

$$\text{also if } \frac{a_{k+1}}{a_k} \to \lambda \qquad \text{(A-15)}$$

if $\lambda < 1$ $\sum_{m=0}^{\infty} a_m$ converges

if $\lambda > 1$ $\sum_{m=0}^{\infty} a_m$ diverges (A-16)

if $\lambda = 1$ then the test ratio inconclusive.

Term-wise addition :

Two power series may be multiplied term by term

so $\sum_{m=0}^{\infty} a_m (x-x_0)^2 \times \sum_{m=0}^{\infty} b_m (x-x_0)^m$

$$= a_0b_0 + (a_0b_1 + a_1b_0)x + (a_0b_2 + a_1b_1 + a_2b_0)x^2 + \ldots \qquad \text{(A-17)}$$

Shifting summation indices :

This is a technically best explained in terms of typical examples, An index of summation is a dummy and can be changed .

For example:

$$\sum_{m=1}^{\infty} \frac{3^m m^2}{m!} = \sum_{k=1}^{\infty} \frac{3^k k^2}{k!} = 3 + 28 + \frac{81}{2} + \ldots \qquad \text{(A-18)}$$

we can shift an index if series , for example

if we set m = s+2 then s= m-2 and

$$\sum_{m=2}^{\infty} m(m-1) a_m x^{m-2} = \sum_{s=0}^{\infty} (s+2)(s+1) a_{s+2} x^s$$

$$= 2a_2 + 6a_2x + 12a_2x^2 + \ldots.. \qquad \text{(A-19)}$$

This is needed, for instance , in writing the sum of two series as a single series.

Appendix B : Taylor Series in x

If we have a function of continuous at 0 and set $P_0(x) = f(0)$, if f is a differentiable at 0, the linear function that best approximates f at points close to 0 is the function

$$P_1(x) = f(0) + f'(0)x \quad \text{(B-1)}$$

P_1 has the same value as f at 0 an also the same first derivative (the same rate of change) $P_1(0) = f(0) \qquad P_1'(0) = f'(0)$ (B-2)

if f has two derivatives at 0, then we can get a better approximation to f by using the quadratic polynomial

$$P_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 \quad \text{(B-3)}$$

P_2 has the same value as f at 0 and the same first two derivatives

$$P_2(0) = f(0), P_2'(0) = f'(0), P_2''(0) = f''(0) \quad \text{(B-4)}$$

and so on.

more generally, if f has n derivatives at 0, we can form the polynomial

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^n(0)}{n!}x^n \quad \text{(B-5)}$$

the approximating polynomials,

$P_0(x)$, $P_1(x)$, $P_2(x)$,....., $P_n(x)$ are called Taylor polynomials.

if f has n+1 continuous derivatives on the interval I that joins 0 to x then

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^n(0)}{n!}x^n + R_{n+1}(x) \quad \text{(B-6)}$$

where the remainder $R_{n+1}(x)$ is given by the formula

$$R_{n+1}(x) = \frac{1}{n!}\int_0^1 f^{n+1}(t)(x-t)^n dt \qquad \text{(B-7)}$$

if $R_{n+1}(x) \to 0$ then

$$f(k) = \sum_{k=0}^{\infty} \frac{f^k(0)}{k!} x^k \qquad \text{(B-8)}$$

Appendix C: Formulation of non linear terms

if $W(x) = \sum_{n=0}^{\infty} a_n x^n$

$$= a_0 + a_1 x + a_2 x^2 + ... \quad \text{(C-1)}$$

then

$$W^3(x) = (\sum_{n=0}^{\infty} a_n x^n)^3$$

$$= (a_0 + a_1 x + a_2 x^2 + ...)^3 \quad \text{(C-2)}$$

$$= (a_0 a_0 + a_0 a_1 x + a_0 a_2 x^2 + a_0 a_3 x^3 + ...$$
$$+ a_1 a_0 x + a_1 a_1 x^2 + a_1 a_2 x^3 + a_1 a_3 x^4 + ...$$
$$+ a_2 a_0 x^2 + a_2 a_1 x^3 + a_2 a_2 x^4 + a_2 a_3 x^5 + ...$$
$$+ a_3 + ...) (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ...) \quad \text{(C-3)}$$

$$= a_0 a_0 a_0 + a_0 a_0 a_1 x + a_0 a_0 a_2 x^2 + a_0 a_0 a_3 x^3 + ...$$
$$+ a_0 a_1 a_0 x + a_0 a_1 a_1 x^2 + a_0 a_1 a_2 x^3 + ...$$
$$+ a_0 a_2 a_0 x^2 + a_2 a_1 a_0 x^3 + ...$$
$$+ a_1 a_0 a_0 x + a_0 a_1 a_2 x^2 + a_0 a_2 a_1 x^3 + ...$$
$$+ a_1 a_0 a_1 x^2 + a_1 a_1 a_1 x^3 + a_2 a_0 a_1 x^3$$
$$+ a_0 a_0 a_2 x^2 + a_0 a_1 a_2 x^3 + a_1 a_0 a_2 x^3 + ... \quad \text{(C-4)}$$

this yields to

$$A_0 + A_1\,x + A_2\,x^2 + A_3\,x^3 + \ldots\ldots\ldots \qquad \text{(C-5)}$$

where :

$$A_0 = a_0\,a_0\,a_0 = a_0^{\,3}$$

$$A_1 = \begin{pmatrix} & a_0 & a_0 & a_0 \\ + & a_0 & a_1 & a_0 \\ + & a_1 & a_0 & a_0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} & a_0 & a_0 & a_2 \\ + & a_0 & a_1 & a_1 \\ + & a_0 & a_2 & a_0 \\ + & a_1 & a_0 & a_1 \\ + & a_1 & a_1 & a_0 \\ + & a_2 & a_0 & a_0 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} & a_0 & a_0 & a_3 \\ + & a_0 & a_1 & a_2 \\ + & a_0 & a_2 & a_1 \\ + & a_0 & a_3 & a_0 \\ + & a_1 & a_0 & a_2 \\ + & a_1 & a_1 & a_1 \\ + & a_1 & a_2 & a_0 \\ + & a_2 & a_0 & a_1 \\ + & a_2 & a_1 & a_0 \\ + & a_3 & a_0 & a_0 \end{pmatrix}$$

and so on.

and more general

$$A_n = \sum_{i=0}^{n} \sum_{j=0}^{n-i} \left(a_i a_j a_{n-(i+j} \right. \qquad \text{(C-7)}$$

for

$$A_5 = \quad a_0 \begin{pmatrix} & a_0 & a_5 \\ + & a_1 & a_4 \\ + & a_2 & a_3 \\ + & a_3 & a_2 \\ + & a_4 & a_1 \\ + & a_5 & a_0 \end{pmatrix} \quad a_1 \begin{pmatrix} & a_0 & a_4 \\ + & a_1 & a_3 \\ + & a_2 & a_2 \\ + & a_3 & a_1 \\ + & a_4 & a_0 \end{pmatrix} \quad a_2 \begin{pmatrix} & a_0 & a_3 \\ + & a_1 & a_2 \\ + & a_2 & a_1 \\ + & a_3 & a_0 \end{pmatrix}$$

$$a_3 \begin{pmatrix} & a_0 & a_2 \\ + & a_1 & a_1 \\ + & a_2 & a_0 \end{pmatrix} \quad a_4 \begin{pmatrix} & a_0 & a_1 \\ + & a_1 & a_0 \end{pmatrix} \quad a_5 \begin{pmatrix} a_0 & a_0 \end{pmatrix}$$

(C-8)

Appendix D : Computer Programs
Program Number 1

```
C************************************************************
C   THIS PROGRAM IS SET TO SOLVE THE LARGE DEFLECTIONS OF
     BEAMS
C   BY POWER SERIES METHOD  *** MOHAMMAD AL-HUSBAN
C ************************************************************
     DOUBLE PRECISION A(0:60),MAXA2,MAXA3
     DOUBLE PRECISION AA(0:60),END1,P(0:60),TOL
     DOUBLE PRECISION ALFA,BETA,F1,RFN,AB,WNEW,WPNEW,Z,Q1,Q2
     INTEGER NR,NM
C ************************************************************
C    FOR CLAMPED BEAM A0,A1 ARE ZERO
C ************************************************************
     END1 = 1.0D0
     PRINT*,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
     PRINT *, 'Input the values of  P(0)  &  ALFA  &  BETA'
     PRINT *,
     READ *,P(0),ALFA ,BETA
     PRINT*,'P(0) = ',P(0) ,' ALFA  = ',ALFA ,' BETA = ',BETA
     PRINT*,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
     DO 3 L=1,60
  3   P(L) = 0.0D0
     A(0)=0.0D0
     A(1)=0.0D0
C************************************************************
C    A2 & A3 ,THE FIRST APROXIMATIONS ARE TAKEN
C    FROM THE SOLUTION OF LINEAR DIFFERENTIAL EQUATION .
C************************************************************
     MAXA2 = + (0.05D0 * P(0))
     MAXA3 = - (0.070D0 * P(0))
     NR = 0
     A(2)= + (0.030D0*P(0))
     A(3)= - (0.09D0*P(0))
     PRINT *, '*************************************************'
     PRINT*,'THE VALUE OF DEFLECTION  AT THE MID OF THE BEAM IS'
     PRINT *, '*************************************************'
     NM=6
     DO 999 NM=8,60,2
  1   DO 10 N=0,NM
     CALL TNL(N,F1,A)
     AA(N)=F1
     RFN = (N+4) * (N+3) * (N+2) * (N+1)
     AB = ( ALFA * A(N) ) + ( BETA * AA(N) )
     A(N+4)= (P(N) - AB) / RFN
  10  CONTINUE
  IF (P(0) .NE.0.0D0 ) THEN
```

```
      TOL = 1.0D-4 * P(0)
      ELSE
      TOL = 1.0D-4
      ENDIF
      CALL W(NM,END1,WNEW,A)
      CALL WP(NM,END1,WPNEW,A)
      IF ( ABS(WNEW) .LE. TOL .AND. ABS(WPNEW) .LE. TOL ) THEN
      NR = NR + 1
 Z=5.0D-1
      CALL W(NM,Z,Q1,A)
      CALL WDP(NM,Z,Q2,A)
      PRINT 600,'NM',NM,'| W =',Q1,'| WDP =',Q2
 600  FORMAT  (2X,A3,2X,I3,4X,A6,D25.16,2X,A6,D25.16 )
      ELSE
      A(2) =  A(2) + ( 0.0001D0 * P(0) )
      IF ( A(2) .LE. MAXA2 )THEN
      GO TO 1
      ELSE
      A(3) =  A(3) + ( 0.0001D0  * P(0) )
      ENDIF
      IF ( A(3) .LE. MAXA3 )THEN
      A(2)= + (0.03D0 * P(0))
      GO TO 1
      ENDIF
      ENDIF
 9999 IF ( NR .LT. 1 )THEN
      PRINT*,'NO SOLUTION WITHIN THE GIVEN RANGE WITH THE
 FOLLOWING'
      PRINT*,'PARAMETERS',P(0),ALFA,BETA
      ENDIF
  999 CONTINUE
      PRINT*,'END OF SEARCHING IN THE FOLLOWING LIMITS '
      PRINT*,  MAXA3, MAXA2
      END
C ******************************************************************
C     CALCULATION OF W(X)
C ******************************************************************
      SUBROUTINE W(NMW,X,W1,AW)
      DOUBLE PRECISION W1,X,AW(0:60)
      W1=0.0D0
      DO 30 K=0,NMW
      W1= W1 + ( AW(K)*(X**K))
 30   CONTINUE
      RETURN
      END

C******************************************************************
C     CALCULATION OF WP(Y)
```

```
C*******************************************************************
      SUBROUTINE WP(NMP,Y,WW,AWP)
      DOUBLE PRECISION   WW,Y,AWP(0:60)
      WW=0.0D0
      DO 40 K=1,NMP
      WW = WW + (K*AWP(K)*(Y**(K-1)))
 40   CONTINUE
      RETURN
      END
C*******************************************************************
C     CALCULATION OF WDP(Y)
C*******************************************************************
      SUBROUTINE WDP(NMD,YD,WWD,AWDP)
      DOUBLE PRECISION   WWD,YD,AWDP(0:60)
      WWD=0.0D0
      DO 41 K=1,NMD
      WWD = WWD + ((K*(K-1)) * AWDP(K) * (YD**(K-2)))
 41   CONTINUE
      RETURN
      END
C ****************************************************************
C   CALCULATIONA OF AA(N) : NON LINEAR TERMS OF BEAM
EQUATION
C ****************************************************************
      SUBROUTINE TNL(L,F,AT)
      DOUBLE PRECISION  F ,AT(0:60)
      F=0.0D0
      DO 100 I=0,L
      DO 100 J=0,L-I
      K=L-(I+J)
      F = F + ( AT(I) * AT(J) * AT(K) )
 100  CONTINUE
      RETURN
      END
```

Program Number 2

```
C******************************************************************
C    THIS PROGRAM IS SET TO SOLVE THE LARGE DEFLECTIONS OF
BEAMS
C    BY POWER SERIES METHOD  *** MOHAMMAD AL-HUSBAN
C          *** CLAMPED BEAM  --  CONSTANT LOAD  ***
C *****************************************************************
      DOUBLE PRECISION A(0:35),MAXA2,MAXA3,ZZ,QQ1,QQ2,QQ3
      DOUBLE PRECISION AA(0:30),END1,P(0:35),TOL
      DOUBLE PRECISION
ALFA,BETA,F1,RFN,AB,WNEW,WPNEW,Z,Q1,Q2,Q3
      INTEGER NR
C******************************************************************
```

```
C     FOR CLAMPED BEAM A0,A1 ARE ZERO
C*****************************************************************
      END1 = 1.0D0
      PRINT*,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
      PRINT *, 'Input the values of  P(0)  &  ALFA  &  BETA'
      PRINT *,
      READ *, P(0),ALFA ,BETA
      PRINT*,'P(0) = ',P(0) ,' ALFA  = ',ALFA ,' BETA = ',BETA
      PRINT*,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
      DO 3 L=1,35
 3    P(L) = 0.0D0
      A(0)=0.0D0
      A(1)=0.0D0
C*****************************************************************
C     A2 & A3 ,THE FIRST APROXIMATIONS ARE TAKEN
C     FROM THE SOLUTION OF LINEAR DIFFERENTIAL EQUATION .
C*****************************************************************
      MAXA2 = + (0.05D0 * P(0))
      MAXA3 = - (0.070D0 * P(0))
      NR = 0
      A(2)= + (0.030D0*P(0))
      A(3)= - (0.09D0*P(0))
 1    DO 10 N=0,20
      CALL TNL(N,F1,A)
      AA(N)=F1
      RFN = (N+4) * (N+3) * (N+2) * (N+1)
      AB = ( ALFA * A(N) ) + ( BETA * AA(N) )
      A(N+4)= (P(N) - AB) / RFN
 10   CONTINUE
C*****************************************************************
      IF (P(0) .NE.0.0D0 ) THEN
      TOL = 1.0D-4 * P(0)
      ELSE
      TOL = 1.0D-4
      ENDIF
      CALL W(END1,WNEW,A)
      CALL WP(END1,WPNEW,A)
      IF ( ABS(WNEW) .LE. TOL .AND. ABS(WPNEW) .LE. TOL ) THEN
      NR = NR + 1
      PRINT*,' '
      PRINT*,'SOLUTION No.   ',NR
      PRINT*,' '
      PRINT*,'WNEW & WPNEW (at x= 1.0D0 ,A(2),A(3) ) ='
      PRINT*, WNEW,WPNEW,A(2),A(3)
      PRINT*,
      PRINT*,'*******************************************************'
      PRINT*,'THE COEFFICIENTS OF POWER SERIES SOLUTION & NLT
      AA(N)ARE'
```

```
   PRINT*,'*************************************************************'
    DO 20 M=0,20
 20 PRINT 300,'A(',M,') = ',A(M), '| AA(',M,') = ',AA(M)
300 FORMAT (2X,A2,I2,A4,3X,D24.16,2X,A5,I2,A4,3X,D24.16)
PRINT *, '*******************************************************'
    PRINT*,'THE VALUES OF DEFLECTIONS  ALONG THE BEAM ARE'
    PRINT *, '*******************************************************'
    ZZ=0.0D0
    QQ1=0.0D0
    PRINT 600,' X = ', ZZ,'| W = ',QQ1
    Z=1.0D-1
    DO 22 J=1,10
    CALL W(Z,Q1,A)
    PRINT 600,' X = ',Z,'| W = ',Q1
600 FORMAT  (2X,A6,D10.4,5X,A6,D25.16,2X)
    Z=Z+1.0D-1
 22  CONTINUE
C     *************************************************************
    PRINT *, '*******************************************************'
    PRINT*,'THE VALUES OF SLOPE AND 2ND DERV.W'' ALONG THE
BEAM ARE'
    PRINT *, '*******************************************************'
    QQ2=0.0D0
    QQ3=2.0*A(2)
    PRINT 601,'| WP =',QQ2,'| WDP =',QQ3
    Z=1.0D-1
    DO 23 J=1,10
    CALL WP(Z,Q2,A)
    CALL WDP(Z,Q3,A)
    PRINT 601,'| WP = ',Q2,'| WDP = ',Q3
601 FORMAT  (2X,A6,D25.16,5X,A6,D25.16,2X)
    Z=Z+1.0D-1
 23  CONTINUE
    PRINT*,'---------------------------------------------------'

C ****************************************************************
    GO TO 9999
    ELSE
    ENDIF
    A(2) =  A(2) + ( 0.0001D0 * P(0) )
    IF ( A(2) .LE. MAXA2 )THEN
    GO TO 1
    ELSE
    A(3) =  A(3) + ( 0.0001D0  * P(0) )
    ENDIF
    IF ( A(3) .LE. MAXA3 )THEN
    A(2)= + (0.03D0 * P(0))
    GO TO 1
```

```
      ENDIF
 9999 IF ( NR .LT. 1 )THEN
      PRINT*,'NO SOLUTION WITHIN THE GIVEN RANGE WITH THE
FOLLOWING'
      PRINT*,'PARAMETERS',P(0),ALFA,BETA
      ENDIF
      PRINT*,'END OF SEARCHING IN THE FOLLOWING LIMITS '
      PRINT*,  MAXA3, MAXA2,'No.OF SOLUTIOS ',NR
      END

C*****************************************************************
C     CALCULATION OF W(X)
C*****************************************************************
      SUBROUTINE W(X,W1,AW)
      DOUBLE PRECISION W1,X,AW(0:35)
      W1=0.0D0
      DO 30 K=0,20
      W1= W1 + ( AW(K)*(X**K))
30    CONTINUE
      RETURN
      END

C*****************************************************************
C     CALCULATION OF WP(Y)
C*****************************************************************
      SUBROUTINE WP(Y,WW,AWP)
      DOUBLE PRECISION   WW,Y,AWP(0:35)
      WW=0.0D0
      DO 40 K=1,20
      WW = WW + (K*AWP(K)*(Y**(K-1)))
 40   CONTINUE
      RETURN
      END

C*****************************************************************
C     CALCULATION OF WDP(Y)
C*****************************************************************
      SUBROUTINE WDP(YD,WWD,AWDP)
      DOUBLE PRECISION   WWD,YD,AWDP(0:35)
      WWD=0.0D0
      DO 41 K=1,20
      WWD = WWD + ((K*(K-1)) * AWDP(K) * (YD**(K-2)))
 41   CONTINUE
      RETURN
      END
C ***************************************************************
C   CALCULATIONA OF AA(N) : NON LINEAR TERMS OF BEAM
EQUATION
```

```
C ***********************************************************
      SUBROUTINE TNL(L,F,AT)
      DOUBLE PRECISION  F ,AT(0:35)
      F=0.0D0
      DO 100 I=0,L
      DO 100 J=0,L-I
      K=L-(I+J)
      F = F + ( AT(I) * AT(J) * AT(K) )
100   CONTINUE
      RETURN
      END
```

Program Number 3

```
C ***************************************************************
C   THIS PROGRAM IS SET TO SOLVE THE LARGE DEFLECTIONS OF
BEAMS
C   BY POWER SERIES METHOD  *** MOHAMMAD AL-HUSBAN
C        **** CLAMPED BEAM  --  CUBIC  LOAD  ****
C***************************************************************
      DOUBLE PRECISION A(0:35),MAXA2,MAXA3,ZZ,QQ1,QQ2,QQ3
      DOUBLE PRECISION AA(0:30),END1,P(0:35),TOL
      DOUBLE PRECISION
      ALFA,BETA,F1,RFN,AB,WNEW,WPNEW,Z,Q1,Q2,Q3
      INTEGER NR
C***************************************************************
C    FOR CLAMPED BEAM A0,A1 ARE ZERO
C***************************************************************
      END1 = 1.0D0
      PRINT *,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
      PRINT *, 'THE LOAD IS QUADRATIC ( NOT SYMETRIC )IN THE FORM'
      PRINT *, ' X**3 - 2*X**2 + X '
      PRINT *,
      READ *, ALFA ,BETA
      PRINT*,' ALFA  = ',ALFA ,' BETA = ',BETA
      PRINT*,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
      P(0) = 0.0D0
      P(1) = 1.0D0
      P(2) = -2.0D0
      P(3) = 1.0D0
      DO 3 L=4,35
 3    P(L) = 0.0D0
      A(0)=0.0D0
      A(1)=0.0D0

C***************************************************************
C    A2 & A3 ,THE FIRST APROXIMATIONS ARE TAKEN
C    FROM THE SOLUTION OF LINEAR DIFFERENTIAL EQUATION .
```

```
C*****************************************************************
      MAXA2 = + (0.0055D0 )
      MAXA3 = - (0.008D0 )
      NR = 0
      A(2)=  (0.004D0)
      A(3)= - (0.009D0)

1     DO 10 N=0,20
      CALL TNL(N,F1,A)
      AA(N)=F1
      RFN = (N+4) * (N+3) * (N+2) * (N+1)
      AB = ( ALFA * A(N) ) + ( BETA * AA(N) )
      A(N+4) = (P(N) - AB) / RFN
 10   CONTINUE
C*****************************************************************
      TOL = 1.0D-5
      CALL W(END1,WNEW,A)
      CALL WP(END1,WPNEW,A)
      IF ( ABS(WNEW) .LE. TOL .AND. ABS(WPNEW) .LE. TOL ) THEN
      NR = NR + 1
      PRINT*,' '
      PRINT*,'SOLUTION No.   ',NR
      PRINT*,' '
      PRINT*,'WNEW & WPNEW (at x= 1.0D0 ,A(2),A(3) ) ='
      PRINT*, WNEW,WPNEW,A(2),A(3)
      PRINT*,
      PRINT*,'*********************************************************'
      PRINT*,'THE COEFFICIENTS OF POWER SERIES SOLUTION & NLT
      AA(N)ARE'
      PRINT*,'*********************************************************'
      DO 20 M=0,20
  20  PRINT 300,'A(',M,') = ',A(M),  '| AA(',M,') = ',AA(M)
 300  FORMAT (2X,A2,I2,A4,3X,D24.16,2X,A5,I2,A4,3X,D24.16)
C*****************************************************************
      PRINT *, '*****************************************************'
      PRINT*,'THE VALUES OF DEFLECTIONS  ALONG THE BEAM ARE'
      PRINT *, '*****************************************************'
      ZZ=0.0D0
      QQ1=0.0D0
      PRINT 600,' X = ', ZZ,'| W = ',QQ1
      Z=1.0D-1
      DO 22 J=1,10
      CALL W(Z,Q1,A)
      PRINT 600,' X = ',Z,'| W = ',Q1
 600  FORMAT  (2X,A6,D10.4,5X,A6,D25.16,2X)
      Z=Z+1.0D-1
  22  CONTINUE
C     ***********************************************************
```

```
      PRINT *, '*****************************************************'
      PRINT*,'THE VALUES OF SLOPE AND 2ND DERV.W" ALONG THE
BEAM ARE'
      PRINT *, '*****************************************************'
      QQ2=0.0D0
      QQ3=2.0*A(2)
      PRINT 601,'|WP = ',QQ2,'|WDP = ',QQ3
      Z=1.0D-1
      DO 23 J=1,10
      CALL WP(Z,Q2,A)
      CALL WDP(Z,Q3,A)
      PRINT 601,'|WP = ',Q2,'|WDP = ',Q3
 601  FORMAT  (2X,A6,D25.16,5X,A6,D25.16,2X)
      Z=Z+1.0D-1
 23   CONTINUE
      PRINT*,'-----------------------------------------------------'

C*****************************************************************
      GO TO 9999
      ELSE
      ENDIF
      A(2) =  A(2) + ( 0.00001D0 )
      IF ( A(2) .LE. MAXA2 )THEN
      GO TO 1
      ELSE
      A(3) =  A(3) + ( 0.00001D0 )
      ENDIF
      IF ( A(3) .LE. MAXA3 )THEN
      A(2)= + (0.0D0 )
      GO TO 1
      ENDIF
 9999 IF ( NR .LT. 1 )THEN
      PRINT*,'NO SOLUTION WITHIN THE GIVEN RANGE WITH THE
FOLLOWING'
      PRINT*,'PARAMETERS',ALFA,BETA
      ENDIF
      PRINT*,'END OF SEARCHING IN THE FOLLOWING LIMITS '
      PRINT*,  MAXA3, MAXA2,'No.OF SOLUTIOS ',NR
      END

C*****************************************************************
C     CALCULATION OF W(X)
C*****************************************************************
      SUBROUTINE W(X,W1,AW)
      DOUBLE PRECISION W1,X,AW(0:35)
      W1=0.0D0
      DO 30 K=0,20
      W1= W1 + ( AW(K)*(X**K))
```

```
30   CONTINUE
     RETURN
     END

C
*******************************************************************
C    CALCULATION OF WP(Y)
C*******************************************************************
     SUBROUTINE WP(Y,WW,AWP)
     DOUBLE PRECISION   WW,Y,AWP(0:35)
     WW=0.0D0
     DO 40 K=1,20
     WW = WW + (K*AWP(K)*(Y**(K-1)))
 40  CONTINUE
     RETURN
     END

C*******************************************************************
C    CALCULATION OF WDP(Y)
C*******************************************************************
     SUBROUTINE WDP(YD,WWD,AWDP)
     DOUBLE PRECISION   WWD,YD,AWDP(0:35)
     WWD=0.0D0
     DO 41 K=1,20
     WWD = WWD + ((K*(K-1)) * AWDP(K) * (YD**(K-2)))
 41  CONTINUE
     RETURN
     END
C ***************************************************************
C  CALCULATIONA OF AA(N) : NON LINEAR TERMS OF BEAM
EQUATION
C ***************************************************************
     SUBROUTINE TNL(L,F,AT)
     DOUBLE PRECISION  F ,AT(0:35)
     F=0.0D0
     DO 100 I=0,L
     DO 100 J=0,L-I
     K=L-(I+J)
     F = F + ( AT(I) * AT(J) * AT(K) )
 100 CONTINUE
     RETURN
     END
```

Program Number 4

```
C *****************************************************************
```

```
C   THIS PROGRAM IS SET TO SOLVE THE LARGE DEFLECTIONS OF
BEAMS
C   BY POWER SERIES METHOD  *** MOHAMMAD AL-HUSBAN
C         *** CLAMPED BEAM -- SINUSOIDAL LOAD   ***
C ****************************************************************
      DOUBLE PRECISION A(0:35),MAXA2,MAXA3,ZZ,QQ1,QQ2,QQ3
      DOUBLE PRECISION AA(0:30),END1,P(0:35),TOL,FAC,BI
      DOUBLE PRECISION
ALFA,BETA,F1,RFN,AB,WNEW,WPNEW,Z,Q1,Q2,Q3
      INTEGER NR
C****************************************************************
C     FOR CLAMPED BEAM A0,A1 ARE ZERO
C****************************************************************
      END1 = 1.0D0
      PRINT *,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
      PRINT *, 'THE LOAD IS  HALF SINE WAVE ( SYMETRIC )IN THE
FORM'
      PRINT *, ' SINE BI X '
      PRINT *,
      READ *, ALFA ,BETA
      PRINT*,' ALFA  = ',ALFA ,' BETA = ',BETA
      PRINT*,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
      BI = 22.0D0 / 7.0D0
      DO 3 L=0,15
      NN=2*L+1
      CALL FACT(FAC,NN)
      P(2*L) = 0.0D0
    3 P(2*L+1) = (((-1d0)**L)*(BI**(2*L+1)))  / (FAC)
      A(0)=0.0D0
      A(1)=0.0D0

C****************************************************************
C     A2 & A3 ,THE FIRST APROXIMATIONS ARE TAKEN
C     FROM THE SOLUTION OF LINEAR DIFFERENTIAL EQUATION .
C****************************************************************
      MAXA2 = + 0.04D0
      MAXA3 = - 0.03D0
      NR = 0
      A(2)= + 0.01D0
      A(3)= - 0.08D0

 1    DO 10 N=0,20
      CALL TNL(N,F1,A)
      AA(N)=F1
      RFN = (N+4) * (N+3) * (N+2) * (N+1)
      AB = ( ALFA * A(N) ) + ( BETA * AA(N) )
      A(N+4) = (P(N) - AB) / RFN
   10 CONTINUE
```

```
C*********************************************************************
      TOL = 1.0D-4
      CALL W(END1,WNEW,A)
      CALL WP(END1,WPNEW,A)
      IF ( ABS(WNEW) .LE. TOL .AND. ABS(WPNEW) .LE. TOL ) THEN
      NR = NR + 1
      PRINT*,' '
      PRINT*,'SOLUTION No.   ',NR
      PRINT*,' '
      PRINT*,'WNEW & WPNEW (at x= 1.0D0 ,A(2),A(3) ) ='
      PRINT*, WNEW,WPNEW,A(2),A(3)
      PRINT*,
      PRINT*,'***********************************************************'
      PRINT*,'THE COEFFICIENTS OF POWER SERIES SOLUTION & NLT
AA(N)ARE'
      PRINT*,'***********************************************************'
      DO 20 M=0,20
   20 PRINT 300,'A(',M,') = ',A(M), '| AA(',M,') = ',AA(M)
  300 FORMAT (2X,A2,I2,A4,3X,D24.16,2X,A5,I2,A4,3X,D24.16)

C*********************************************************************
      PRINT *, '*******************************************************'
      PRINT*,'THE VALUES OF DEFLECTIONS  ALONG THE BEAM ARE'
      PRINT *, '*******************************************************'
      ZZ=0.0D0
      QQ1=0.0D0
      PRINT 600,' X = ', ZZ,'| W = ',QQ1
      Z=1.0D-1
      DO 22 J=1,10
      CALL W(Z,Q1,A)
      PRINT 600,' X = ',Z,'| W = ',Q1
  600 FORMAT  (2X,A6,D10.4,5X,A6,D25.16,2X)
      Z=Z+1.0D-1
   22 CONTINUE
C     ****************************************************************
      PRINT *, '*********************************************************'
      PRINT*,'THE VALUES OF SLOPE AND 2ND DERV.W" ALONG THE
BEAM ARE'
      PRINT *, '*********************************************************'
      QQ2=0.0D0
      QQ3=2.0*A(2)
      PRINT 601,'|WP = ',QQ2,'|WDP = ',QQ3
      Z=1.0D-1
      DO 23 J=1,10
      CALL WP(Z,Q2,A)
      CALL WDP(Z,Q3,A)
      PRINT 601,'|WP = ',Q2,'|WDP = ',Q3
  601 FORMAT  (2X,A6,D25.16,5X,A6,D25.16,2X)
```

```
      Z=Z+1.0D-1
 23   CONTINUE
      PRINT*,'----------------------------------------------------'

C****************************************************************
      GO TO 9999
      ELSE
      ENDIF
      A(2) =  A(2) + ( 0.001D0  )
      IF ( A(2) .LE. MAXA2 )THEN
      GO TO 1
      ELSE
      A(3) =  A(3) + ( 0.0001D0  )
      ENDIF
      IF ( A(3) .LE. MAXA3 )THEN
      A(2)= + (0.01D0 )
      GO TO 1
      ENDIF
 9999 IF ( NR .LT. 1 )THEN
      PRINT*,'NO SOLUTION WITHIN THE GIVEN RANGE WITH THE
FOLLOWING'
      PRINT*,'PARAMETERS',ALFA,BETA
      ENDIF
      PRINT*,'END OF SEARCHING IN THE FOLLOWING LIMITS '
      PRINT*,  MAXA3, MAXA2,'No.OF SOLUTIOS ',NR
      END

C****************************************************************
C     CALCULATION OF FACTORIAL OF INTEGER
C****************************************************************
      SUBROUTINE FACT(F,M)
      DOUBLE PRECISION F
      IF (M.EQ.0) THEN
      F = 1.0D0
      ELSE
      F=1.0D0
      DO 4 L=1,M
  4   F = F*L
      ENDIF
      RETURN
      END

C****************************************************************
C     CALCULATION OF W(X)
C****************************************************************
      SUBROUTINE W(X,W1,AW)
      DOUBLE PRECISION W1,X,AW(0:35)
      W1=0.0D0
```

```
      DO 30 K=0,20
      W1= W1 + ( AW(K)*(X**K))
30    CONTINUE
      RETURN
      END

C*****************************************************************
C     CALCULATION OF WP(Y)
C*****************************************************************
      SUBROUTINE WP(Y,WW,AWP)
      DOUBLE PRECISION   WW,Y,AWP(0:35)
      WW=0.0D0
      DO 40 K=1,20
      WW = WW + (K*AWP(K)*(Y**(K-1)))
40    CONTINUE
      RETURN
      END

C*****************************************************************
C     CALCULATION OF WDP(Y)
C*****************************************************************
      SUBROUTINE WDP(YD,WWD,AWDP)
      DOUBLE PRECISION   WWD,YD,AWDP(0:35)
      WWD=0.0D0
      DO 41 K=1,20
      WWD = WWD + ((K*(K-1)) * AWDP(K) * (YD**(K-2)))
41    CONTINUE
      RETURN
      END


C ***********************************************************
C   CALCULATIONA OF AA(N) : NON LINEAR TERMS OF BEAM
EQUATION
C ***********************************************************
      SUBROUTINE TNL(L,F,AT)
      DOUBLE PRECISION  F ,AT(0:35)
      F=0.0D0
      DO 100 I=0,L
      DO 100 J=0,L-I
      K=L-(I+J)
      F = F + ( AT(I) * AT(J) * AT(K) )
100   CONTINUE
      RETURN
      END
```

Program Number 5

```
C*****************************************************************
C   THIS PROGRAM IS SET TO SOLVE THE LARGE DEFLECTIONS OF
BEAMS
C   BY POWER SERIES METHOD  *** MOHAMMAD AL-HUSBAN
C       *** SIMPLY SUPPORTED BEAM  --  CONSTANT LOAD  ***
C*****************************************************************

      DOUBLE PRECISION A(0:25),MAXA1,MAXA3,ZZ,QQ1,QQ2,QQ3
      DOUBLE PRECISION AA(0:25),END1,P(0:25),TOL
      DOUBLE PRECISION
ALFA,BETA,F1,RFN,AB,WNEW,WDPNEW,Z,Q1,Q2,Q3
      INTEGER NR
C*****************************************************************
C     FOR CLAMPED BEAM A0,A1 ARE ZERO
C*****************************************************************
      END1 = 1.0D0
      PRINT*,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
      PRINT *, 'Input the values of  P(0)  &  ALFA  &  BETA'
      PRINT *,
      READ *, P(0),ALFA ,BETA
      PRINT*,'P(0) = ',P(0) ,' ALFA  = ',ALFA ,' BETA = ',BETA
      PRINT*,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
      DO 3 L=1,25
 3    P(L) = 0.0D0
      A(0)=0.0D0
      A(2)=0.0D0

C*****************************************************************
C     A1 & A3 ,THE FIRST APROXIMATIONS ARE TAKEN
C     FROM THE SOLUTION OF LINEAR DIFFERENTIAL EQUATION .
C*****************************************************************
      MAXA1 = + (0.06D0 * P(0))
      MAXA3 = - (0.06D0 * P(0))
      NR = 0
      A(1)= + (0.030D0*P(0))
      A(3)= - (0.1D0*P(0))

 1    DO 10 N=0,20
      CALL TNL(N,F1,A)
      AA(N)=F1
      RFN = (N+4) * (N+3) * (N+2) * (N+1)
      AB = ( ALFA * A(N) ) + ( BETA * AA(N) )
      A(N+4)= (P(N) - AB) / RFN
 10   CONTINUE
C*****************************************************************
      IF (P(0) .NE.0.0D0 ) THEN
      TOL = 1.0D-4 * P(0)
      ELSE
```

```
      TOL = 1.0D-4
      ENDIF
      CALL W(END1,WNEW,A)
      CALL WDP(END1,WDPNEW,A)
      IF ( ABS(WNEW) .LE. TOL .AND. ABS(WDPNEW) .LE. TOL ) THEN
      NR = NR + 1
      PRINT*,' '
      PRINT*,'SOLUTION No.   ',NR
      PRINT*,' '
      PRINT*,'WNEW & WDPNEW (at x= 1.0D0 ,A(1),A(3) ) ='
      PRINT*, WNEW,WDPNEW,A(1),A(3)
      PRINT*,
      PRINT*,'***********************************************************'
      PRINT*,'THE COEFFICIENTS OF POWER SERIES SOLUTION & NLT
AA(N)ARE'
      PRINT*,'***********************************************************'
      DO 20 M=0,20
   20 PRINT 300,'A(',M,') = ',A(M), '| AA(',M,') = ',AA(M)
  300 FORMAT (2X,A2,I2,A4,3X,D24.16,2X,A5,I2,A4,3X,D24.16)

C*****************************************************************
      PRINT *, '*******************************************************'
      PRINT*,'THE VALUES OF DEFLECTIONS  ALONG THE BEAM ARE'
      PRINT *, '*******************************************************'
      ZZ=0.0D0
      QQ1=0.0D0
      PRINT 600,' X = ', ZZ,'| W = ',QQ1
      Z=1.0D-1
      DO 22 J=1,10
      CALL W(Z,Q1,A)
      PRINT 600,' X = ',Z,'| W = ',Q1
  600 FORMAT  (2X,A6,D10.4,5X,A6,D25.16,2X)
      Z=Z+1.0D-1
   22 CONTINUE
C     *************************************************************
      PRINT *, '*********************************************************'
      PRINT*,'THE VALUES OF SLOPE AND 2ND DERV.W" ALONG THE
BEAM ARE'
      PRINT *, '*******************************************************'
      QQ2=A(1)
      QQ3=2.0 * A(2)
      PRINT 601,'| WP =',QQ2,'| WDP =',QQ3
      Z=1.0D-1
      DO 23 J=1,10
      CALL WP(Z,Q2,A)
      CALL WDP(Z,Q3,A)
      PRINT 601,'| WP = ',Q2,'| WDP = ',Q3
  601 FORMAT  (2X,A6,D25.16,5X,A6,D25.16,2X)
```

```
      Z=Z+1.0D-1
 23   CONTINUE
      PRINT*,'------------------------------------------------------'

C*******************************************************************
      GO TO 9999
      ELSE
      ENDIF
      A(1) =  A(1) + ( 0.0001D0 * P(0) )
      IF ( A(1) .LE. MAXA1 )THEN
      GO TO 1
      ELSE
      A(3) =  A(3) + ( 0.0001D0  * P(0) )
      ENDIF
      IF ( A(3) .LE. MAXA3 )THEN
      A(1)= + (0.03D0 * P(0))
      GO TO 1
      ENDIF
 9999 IF ( NR .LT. 1 )THEN
      PRINT*,'NO SOLUTION WITHIN THE GIVEN RANGE WITH THE
FOLLOWING'
      PRINT*,'PARAMETERS',P(0),ALFA,BETA
      ENDIF
      PRINT*,'END OF SEARCHING IN THE FOLLOWING LIMITS '
      PRINT*,  MAXA3, MAXA1,'No.OF SOLUTIOS ',NR
      END

C*******************************************************************
C     CALCULATION OF W(X)
C*******************************************************************
      SUBROUTINE W(X,W1,AW)
      DOUBLE PRECISION W1,X,AW(0:25)
      W1=0.0D0
      DO 30 K=0,20
      W1= W1 + ( AW(K)*(X**K))
 30   CONTINUE
      RETURN
      END

C*******************************************************************
C     CALCULATION OF WP(Y)
C*******************************************************************
      SUBROUTINE WP(Y,WW,AWP)
      DOUBLE PRECISION   WW,Y,AWP(0:25)
      WW=0.0D0
      DO 40 K=1,20
      WW = WW + (K*AWP(K)*(Y**(K-1)))
 40   CONTINUE
```

```
      RETURN
      END

C*****************************************************************
C     CALCULATION OF WDP(Y)
C*****************************************************************
      SUBROUTINE WDP(YD,WWD,AWDP)
      DOUBLE PRECISION   WWD,YD,AWDP(0:25)
      WWD=0.0D0
      DO 41 K=1,20
      WWD = WWD + ((K*(K-1)) * AWDP(K) * (YD**(K-2)))
 41   CONTINUE
      RETURN
      END
C ***************************************************************
C   CALCULATIONA OF AA(N) : NON LINEAR TERMS OF BEAM
EQUATION
C ***************************************************************
      SUBROUTINE TNL(L,F,AT)
      DOUBLE PRECISION  F ,AT(0:25)
      F=0.0D0
      DO 100 I=0,L
      DO 100 J=0,L-I
      K=L-(I+J)
      F = F + ( AT(I) * AT(J) * AT(K) )
 100  CONTINUE
      RETURN
      END
```

Program Number 6

```
C*****************************************************************
C   THIS PROGRAM IS SET TO SOLVE THE LARGE DEFLECTIONS OF
BEAMS
C   BY POWER SERIES METHOD  *** MOHAMMAD AL-HUSBAN
C        **** SIMPLY SUPPORTED BEAM  --  CUBIC  LOAD  ****
C*****************************************************************

      DOUBLE PRECISION A(0:35),MAXA1,MAXA3,ZZ,QQ1,QQ2,QQ3
      DOUBLE PRECISION AA(0:30),END1,P(0:35),TOL
      DOUBLE PRECISION
ALFA,BETA,F1,RFN,AB,WNEW,WDPNEW,Z,Q1,Q2,Q3
      INTEGER NR
C*****************************************************************
C    FOR CLAMPED BEAM A0,A1 ARE ZERO
C*****************************************************************
      END1 = 1.0D0
      PRINT *,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
```

```
      PRINT *, 'THE LOAD IS CUBIC ( NOT SYMETRIC )IN THE FORM'
      PRINT *, ' X**3 - 2*X**2 + X '
      PRINT *,
      READ *, ALFA ,BETA
      PRINT*,' ALFA  = ',ALFA ,' BETA = ',BETA
      PRINT*,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
      P(0) = 0.0D0
      P(1) = 1.0D0
      P(2) = -2.0D0
      P(3) = 1.0D0
      DO 3 L=4,35
 3    P(L) = 0.0D0
      A(0)=0.0D0
      A(2)=0.0D0

C*****************************************************************
C     A1 & A3 ,THE FIRST APROXIMATIONS ARE TAKEN
C     FROM THE SOLUTION OF LINEAR DIFFERENTIAL EQUATION .
C*****************************************************************
      MAXA1 = + (0.0055D0 )
      MAXA3 = - (0.008D0 )
      NR = 0
      A(1)=  (0.004D0)
      A(3)= - (0.009D0)

 1    DO 10 N=0,20
      CALL TNL(N,F1,A)
      AA(N)=F1
      RFN = (N+4) * (N+3) * (N+2) * (N+1)
      AB = ( ALFA * A(N) ) + ( BETA * AA(N) )
      A(N+4) = (P(N) - AB) / RFN
 10   CONTINUE
C*****************************************************************
      TOL = 1.0D-4
      CALL W(END1,WNEW,A)
      CALL WDP(END1,WDPNEW,A)
      IF ( ABS(WNEW) .LE. TOL .AND. ABS(WDPNEW) .LE. TOL ) THEN
      NR = NR + 1
      PRINT*,' '
      PRINT*,'SOLUTION No.   ',NR
      PRINT*,' '
      PRINT*,'WNEW & WDPNEW (at x= 1.0D0 ,A(1),A(3) ) ='
      PRINT*, WNEW,WDPNEW,A(1),A(3)
      PRINT*,
      PRINT*,'*********************************************************'
      PRINT*,'THE COEFFICIENTS OF POWER SERIES SOLUTION & NLT
AA(N)ARE'
      PRINT*,'*********************************************************'
```

```
      DO 20 M=0,20
 20  PRINT 300,'A(',M,') = ',A(M),  '| AA(',M,') = ',AA(M)
300  FORMAT (2X,A2,I2,A4,3X,D24.16,2X,A5,I2,A4,3X,D24.16)

C*****************************************************************
      PRINT *, '******************************************************'
      PRINT*,'THE VALUES OF DEFLECTIONS  ALONG THE BEAM ARE'
      PRINT *, '******************************************************'
      ZZ=0.0D0
      QQ1=0.0D0
      PRINT 600,' X = ', ZZ,'| W = ',QQ1
      Z=1.0D-1
      DO 22 J=1,10
      CALL W(Z,Q1,A)
      PRINT 600,' X = ',Z,'| W = ',Q1
 600  FORMAT  (2X,A6,D10.4,5X,A6,D25.16,2X)
      Z=Z+1.0D-1
 22   CONTINUE
C     *************************************************************
      PRINT *, '******************************************************'
      PRINT*,'THE VALUES OF SLOPE AND 2ND DERV.W" ALONG THE
BEAM ARE'
      PRINT *, '******************************************************'
      QQ2=A(1)
      QQ3=0.0D0
      PRINT 601,'|WP = ',QQ2,'|WDP = ',QQ3
      Z=1.0D-1
      DO 23 J=1,10
      CALL WP(Z,Q2,A)
      CALL WDP(Z,Q3,A)
      PRINT 601,'|WP = ',Q2,'|WDP = ',Q3
 601  FORMAT  (2X,A6,D25.16,5X,A6,D25.16,2X)
      Z=Z+1.0D-1
 23   CONTINUE
      PRINT*,'------------------------------------------------------'

C*****************************************************************
      GO TO 9999
      ELSE
      ENDIF
      A(1) =  A(1) + ( 0.00001D0 )
      IF ( A(1) .LE. MAXA1 )THEN
      GO TO 1
      ELSE
      A(3) =  A(3) + ( 0.00001D0 )
      ENDIF
      IF ( A(3) .LE. MAXA3 )THEN
      A(1)= + (0.0D0 )
```

```
      GO TO 1
      ENDIF
 9999 IF ( NR .LT. 1 )THEN
      PRINT*,'NO SOLUTION WITHIN THE GIVEN RANGE WITH THE
FOLLOWING'
      PRINT*,'PARAMETERS',ALFA,BETA
      ENDIF
      PRINT*,'END OF SEARCHING IN THE FOLLOWING LIMITS '
      PRINT*,  MAXA3, MAXA1,'No.OF SOLUTIOS ',NR
      END

C***************************************************************
C     CALCULATION OF W(X)
C***************************************************************
      SUBROUTINE W(X,W1,AW)
      DOUBLE PRECISION W1,X,AW(0:35)
      W1=0.0D0
      DO 30 K=0,20
      W1= W1 + ( AW(K)*(X**K))
 30   CONTINUE
      RETURN
      END

C***************************************************************
C     CALCULATION OF WP(Y)
C***************************************************************
      SUBROUTINE WP(Y,WW,AWP)
      DOUBLE PRECISION   WW,Y,AWP(0:35)
      WW=0.0D0
      DO 40 K=1,20
      WW = WW + (K*AWP(K)*(Y**(K-1)))
 40   CONTINUE
      RETURN
      END

C***************************************************************
C     CALCULATION OF WDP(Y)
C***************************************************************
      SUBROUTINE WDP(YD,WWD,AWDP)
      DOUBLE PRECISION   WWD,YD,AWDP(0:35)
      WWD=0.0D0
      DO 41 K=1,20
      WWD = WWD + ((K*(K-1)) * AWDP(K) * (YD**(K-2)))
 41   CONTINUE
      RETURN
      END
C ***************************************************************
```

```
C  CALCULATIONA OF AA(N) : NON LINEAR TERMS OF BEAM
EQUATION
C ***************************************************************
     SUBROUTINE TNL(L,F,AT)
     DOUBLE PRECISION  F ,AT(0:35)
     F=0.0D0
     DO 100 I=0,L
     DO 100 J=0,L-I
     K=L-(I+J)
     F = F + ( AT(I) * AT(J) * AT(K) )
 100  CONTINUE
     RETURN
     END
```

Program Number 7

```
C ***************************************************************
C   THIS PROGRAM IS SET TO SOLVE THE LARGE DEFLECTIONS OF
BEAMS
C   BY POWER SERIES METHOD  *** MOHAMMAD AL-HUSBAN
C      *** SIMPLY SUPPORTED BEAM -- SINUSOIDAL LOAD   ***
C***************************************************************

     DOUBLE PRECISION A(0:35),MAXA1,MAXA3,ZZ,QQ1,QQ2,QQ3
     DOUBLE PRECISION AA(0:30),END1,P(0:35),TOL,TOL1,FAC,BI
     DOUBLE PRECISION
ALFA,BETA,F1,RFN,AB,WNEW,WDPNEW,Z,Q1,Q2,Q3
     INTEGER NR
C***************************************************************
C    FOR CLAMPED BEAM A0,A1 ARE ZERO
C***************************************************************
     END1 = 1.0D0
     PRINT *,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
     PRINT *, 'THE LOAD IS  HALF SINE WAVE ( SYMETRIC )IN THE
FORM'
     PRINT *, ' SINE BI X '
     PRINT *,
     READ *, ALFA ,BETA
     PRINT*,' ALFA  = ',ALFA ,' BETA = ',BETA
     PRINT*,' ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^ '
     BI = 22.0D0 / 7.0D0
     DO 3 L=0,15
     NN=2*L+1
     CALL FACT(FAC,NN)
     P(2*L) = 0.0D0
   3  P(2*L+1) = (((-1d0)**L)*(BI**(2*L+1)))  / (FAC)
     A(0)=0.0D0
     A(2)=0.0D0
```

```
C*****************************************************************
C    A1 & A3 ,THE FIRST APROXIMATIONS ARE TAKEN
C    FROM THE SOLUTION OF LINEAR DIFFERENTIAL EQUATION .
C*****************************************************************
      MAXA1 = + 0.05D0
      MAXA3 = + 0.01D0
      NR = 0
      A(1)= - 0.01D0
      A(3)= - 0.07D0

1     DO 10 N=0,20
      CALL TNL(N,F1,A)
      AA(N)=F1
      RFN = (N+4) * (N+3) * (N+2) * (N+1)
      AB = ( ALFA * A(N) ) + ( BETA * AA(N) )
      A(N+4) = (P(N) - AB) / RFN
 10   CONTINUE
C*****************************************************************
      TOL = 1.0D-4
      TOL1 =1.0D-4
      CALL W(END1,WNEW,A)
      CALL WDP(END1,WDPNEW,A)
      IF ( ABS(WNEW) .LE. TOL .AND. ABS(WDPNEW) .LE. TOL1 ) THEN
      NR = NR + 1
      PRINT*,' '
      PRINT*,'SOLUTION No.   ',NR
      PRINT*,' '
      PRINT*,'WNEW & WDPNEW (at x= 1.0D0 ,A(1),A(3) ) ='
      PRINT*, WNEW,WDPNEW,A(1),A(3)
      PRINT*,
      PRINT*,'*******************************************************'
      PRINT*,'THE COEFFICIENTS OF POWER SERIES SOLUTION & NLT
 AA(N)ARE'

  PRINT*,'*******************************************************'
      DO 20 M=0,20
  20  PRINT 300,'A(',M,') = ',A(M),  '| AA(',M,') = ',AA(M)
 300  FORMAT (2X,A2,I2,A4,3X,D24.16,2X,A5,I2,A4,3X,D24.16)

C *****************************************************************
      PRINT *, '*******************************************************'
      PRINT*,'THE VALUES OF DEFLECTIONS  ALONG THE BEAM ARE'
      PRINT *, '*******************************************************'
      ZZ=0.0D0
      QQ1=0.0D0
      PRINT 600,' X = ', ZZ,'| W = ',QQ1
      Z=1.0D-1
```

```
      DO 22 J=1,10
      CALL W(Z,Q1,A)
      PRINT 600,' X = ',Z,'| W = ',Q1
 600  FORMAT  (2X,A6,D10.4,5X,A6,D25.16,2X)
      Z=Z+1.0D-1
 22   CONTINUE
C     ************************************************************
      PRINT *, '*****************************************************'
      PRINT*,'THE VALUES OF SLOPE AND 2ND DERV.W" ALONG THE
BEAM ARE'
      PRINT *, '*****************************************************'
      QQ2=A(1)
      QQ3=0.0D0
      PRINT 601,'|WP = ',QQ2,'|WDP = ',QQ3
      Z=1.0D-1
      DO 23 J=1,10
      CALL WP(Z,Q2,A)
      CALL WDP(Z,Q3,A)
      PRINT 601,'|WP = ',Q2,'|WDP = ',Q3
 601  FORMAT  (2X,A6,D25.16,5X,A6,D25.16,2X)
      Z=Z+1.0D-1
 23   CONTINUE
      PRINT*,'------------------------------------------------------'

C**************************************************************
      GO TO 9999
      ELSE
C     PRINT*, WNEW,WDPNEW
C     PRINT*,'A1', A(1),'A3',A(3)
      ENDIF
      A(1) =  A(1) + ( 0.001D0  )
      IF ( A(1) .LE. MAXA1 )THEN
      GO TO 1
      ELSE
      A(3) =  A(3) + ( 0.00001D0  )
      ENDIF
      IF ( A(3) .LE. MAXA3 )THEN
      A(1)= - (0.01D0 )
      GO TO 1
      ENDIF
 9999 IF ( NR .LT. 1 )THEN
      PRINT*,'NO SOLUTION WITHIN THE GIVEN RANGE WITH THE
FOLLOWING'
      PRINT*,'PARAMETERS',ALFA,BETA
      ENDIF
      PRINT*,'END OF SEARCHING IN THE FOLLOWING LIMITS '
      PRINT*,  MAXA3, MAXA1,'No.OF SOLUTIOS ',NR
      END
```

```
C*****************************************************************
C    CALCULATION OF FACTORIAL OF INTEGER
C*****************************************************************
      SUBROUTINE FACT(F,M)
      DOUBLE PRECISION F
      IF (M.EQ.0) THEN
      F = 1.0D0
      ELSE
      F=1.0D0
      DO 4 L=1,M
    4 F = F*L
      ENDIF
      RETURN
      END

C*****************************************************************
C    CALCULATION OF W(X)
C*****************************************************************
      SUBROUTINE W(X,W1,AW)
      DOUBLE PRECISION W1,X,AW(0:35)
      W1=0.0D0
      DO 30 K=0,20
      W1= W1 + ( AW(K)*(X**K))
30    CONTINUE
      RETURN
      END
C*****************************************************************
C    CALCULATION OF WP(Y)
C *****************************************************************
      SUBROUTINE WP(Y,WW,AWP)
      DOUBLE PRECISION   WW,Y,AWP(0:35)
      WW=0.0D0
      DO 40 K=1,20
      WW = WW + (K*AWP(K)*(Y**(K-1)))
 40   CONTINUE
      RETURN
      END

C*****************************************************************
C    CALCULATION OF WDP(Y)
C*****************************************************************
      SUBROUTINE WDP(YD,WWD,AWDP)
      DOUBLE PRECISION   WWD,YD,AWDP(0:35)
      WWD=0.0D0
      DO 41 K=1,20
      WWD = WWD + ((K*(K-1)) * AWDP(K) * (YD**(K-2)))
 41   CONTINUE
```

```
      RETURN
      END
C *************************************************************
C   CALCULATIONA OF AA(N) : NON LINEAR TERMS OF BEAM
EQUATION
C *************************************************************
      SUBROUTINE TNL(L,F,AT)
      DOUBLE PRECISION  F ,AT(0:35)
      F=0.0D0
      DO 100 I=0,L
      DO 100 J=0,L-I
      K=L-(I+J)
      F = F + ( AT(I) * AT(J) * AT(K) )
 100  CONTINUE
      RETURN
      END
```

Appendix E : Sample of OutPut Data

```
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
Input the values of  P(0)  &  ALFA  &  BETA

P(0) =    1.00000000000000     ALFA =    10.0000000000000     BETA =
  10.0000000000000
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^

SOLUTION No.           1
WNEW & WPNEW (at x= 1.0D0 ,A(2),A(3) ) =
  0.579808297188052D-004  0.240566961061118D-005  0.411000000000003D-001
 -0.822999999999998D-001
*******************************************************
THE COEFFICIENTS OF POWER SERIES SOLUTION & NLT AA(N)ARE
*******************************************************
A( 0) =    0.0000000000000000E+00  | AA( 0) =    0.0000000000000000E+00
A( 1) =    0.0000000000000000E+00  | AA( 1) =    0.0000000000000000E+00
A( 2) =    0.4110000000000028E-01  | AA( 2) =    0.0000000000000000E+00
A( 3) =   -0.8229999999999978E-01  | AA( 3) =    0.0000000000000000E+00
A( 4) =    0.4166666666666666E-01  | AA( 4) =    0.0000000000000000E+00
A( 5) =    0.0000000000000000E+00  | AA( 5) =    0.0000000000000000E+00
A( 6) =   -0.1141666666666674E-02  | AA( 6) =    0.6942653100000140E-04
A( 7) =    0.9797619047619021E-03  | AA( 7) =   -0.4170659490000044E-03
A( 8) =   -0.2480158730158730E-03  | AA( 8) =    0.1046297907000004E-02
A( 9) =    0.0000000000000000E+00  | AA( 9) =   -0.1403074266999999E-02
A(10) =    0.2127460586640224E-05  | AA(10) =    0.1054938205749997E-02
A(11) =   -0.7104746916185577E-06  | AA(11) =   -0.4005104320119032E-03
A(12) =   -0.6719545740607161E-06  | AA(12) =    0.1626752282010559E-04
```

A(13) = 0.8176423467365962E-06 | AA(13) = 0.5849900655952379E-04
A(14) = -0.4400040236166487E-06 | AA(14) = -0.3352130439329612E-04
A(15) = 0.1224728042440542E-06 | AA(15) = 0.9561428142502407E-05
A(16) = -0.3570413975742873E-08 | AA(16) = -0.3121900083116678E-06
A(17) = -0.1038456738554979E-07 | AA(17) = -0.7641194918784631E-06
A(18) = 0.4624361167880280E-08 | AA(18) = 0.3025848207519833E-06
A(19) = -0.1041011023687055E-08 | AA(19) = -0.2550058475204927E-07
A(20) = 0.2715517907528471E-10 | AA(20) = -0.3500652421080009E-07
****

THE VALUES OF DEFLECTIONS ALONG THE BEAM ARE

****

X = 0.0000E+00 | W = 0.0000000000000000E+00
X = 0.1000E+00 | W = 0.3328656204962398E-03
X = 0.2000E+00 | W = 0.1052205506232910E-02
X = 0.3000E+00 | W = 0.1813765737664971E-02
X = 0.4000E+00 | W = 0.2372233288664150E-02
X = 0.5000E+00 | W = 0.2580515346086118E-02
X = 0.6000E+00 | W = 0.2389205350551332E-02
X = 0.7000E+00 | W = 0.1846382945315680E-02
X = 0.8000E+00 | W = 0.1097797275930918E-02
X = 0.9000E+00 | W = 0.3873855255077213E-03
X = 0.1000E+01 | W = 0.5798082971880780E-04
****

THE VALUES OF SLOPE AND 2ND DERV.W' ALONG THE BEAM ARE

****

| WP = 0.0000000000000000E+00 | WDP 0.8220000000000055E-01
| WP = 0.5917604826607785E-02 | WDP 0.3781697261294067E-01
| WP = 0.7895554879807377E-02 | WDP 0.3397479554487339E-02
| WP = 0.6926920659497741E-02 | WDP -0.2112754481215863E-01
| WP = 0.3997368167467294E-02 | WDP -0.3583221301974189E-01
| WP = 0.8596323362901665E-04 | WDP -0.4077114696335020E-01
| WP = -0.3832062100371614E-02 | WDP -0.3596476356312730E-01
| WP = -0.6781560654871672E-02 | WDP -0.2139433209887631E-01
| WP = -0.7783714556188188E-02 | WDP 0.2993215810178653E-02
| WP = -0.5853248920962122E-02 | WDP 0.3727070519315206E-01
| WP = 0.2405669610611405E-05 | WDP 0.8150631095277232E-01

END OF SEARCHING IN THE FOLLOWING LIMITS
-0.7000000000000000D-001 0.5000000000000000D-001No.OF SOLUTIOS 1

^^

Input the values of P(0) & ALFA & BETA

P(0) = 10.0000000000000 ALFA = 10.0000000000000 BETA =
10.0000000000000
^^

SOLUTION No. 1

WNEW & WPNEW (at x= 1.0D0 ,A(2),A(3)) =
0.578444233955608D-003 0.179344275769950D-004 0.411000000000000
-0.823000000000000

****
THE COEFFICIENTS OF POWER SERIES SOLUTION & NLT AA(N)ARE
****

A(0) =	0.0000000000000000E+00	AA(0) =	0.0000000000000000E+00
A(1) =	0.0000000000000000E+00	AA(1) =	0.0000000000000000E+00
A(2) =	0.4110000000000001E+00	AA(2) =	0.0000000000000000E+00
A(3) =	-0.8229999999999998E+00	AA(3) =	0.0000000000000000E+00
A(4) =	0.4166666666666667E+00	AA(4) =	0.0000000000000000E+00
A(5) =	0.0000000000000000E+00	AA(5) =	0.0000000000000000E+00
A(6) =	-0.1141666666666667E-01	AA(6) =	0.6942653100000004E-01
A(7) =	0.9797619047619046E-02	AA(7) =	-0.4170659490000001E+00
A(8) =	-0.2480158730158730E-02	AA(8) =	0.1046297907000000E+01
A(9) =	0.0000000000000000E+00	AA(9) =	-0.1403074267000000E+01
A(10) =	-0.1150989371693122E-03	AA(10) =	0.1054938205750000E+01
A(11) =	0.5142276893338144E-03	AA(11) =	-0.4005104320119047E+00
A(12) =	-0.8786344682406071E-03	AA(12) =	0.1626752282010583E-01
A(13) =	0.8176423467365967E-03	AA(13) =	0.5849900655952379E-01
A(14) =	-0.4390705572813978E-03	AA(14) =	-0.3359041345908562E-01
A(15) =	0.1220989634684282E-03	AA(15) =	0.1010239269220856E-01
A(16) =	-0.3523097150152295E-05	AA(16) =	-0.2229330875597477E-02
A(17) =	-0.1038456738554979E-04	AA(17) =	0.3291253293658762E-02
A(18) =	0.4633644337740606E-05	AA(18) =	-0.5367958128671976E-02
A(19) =	-0.1099124059992796E-05	AA(19) =	0.5434308005751873E-02
A(20) =	0.1920239054650524E-06	AA(20) =	-0.3653699583763513E-02

****
THE VALUES OF DEFLECTIONS ALONG THE BEAM ARE
****

X = 0.0000E+00	W = 0.0000000000000000E+00
X = 0.1000E+00	W = 0.3328656204953151E-02
X = 0.2000E+00	W = 0.1052205505606567E-01
X = 0.3000E+00	W = 0.1813765714150335E-01
X = 0.4000E+00	W = 0.2372233021703595E-01
X = 0.5000E+00	W = 0.2580513782489265E-01
X = 0.6000E+00	W = 0.2389199342513309E-01
X = 0.7000E+00	W = 0.1846365679273441E-01
X = 0.8000E+00	W = 0.1097757056932025E-01
X = 0.9000E+00	W = 0.3873057526198537E-02
X = 0.1000E+01	W = 0.5784442339555922E-03

****
THE VALUES OF SLOPE AND 2ND DERV.W' ALONG THE BEAM ARE
****

WP = 0.0000000000000000E+00	WDP 0.8220000000000002E+00
WP = 0.5917604826519231E-01	WDP 0.3781697260536875E+00

| WP = 0.7895554851119846E-01 | WDP 0.3397478398303044E-01
| WP = 0.6926919978773556E-01 | WDP -0.2112756186270890E+00
| WP = 0.3997362710699849E-01 | WDP -0.3583230743062833E+00
| WP = 0.8593932778112696E-03 | WDP -0.4077144781441712E+00
| WP = -0.3832133255126358E-01 | WDP -0.3596543407948383E+00
| WP = -0.6781722966902951E-01 | WDP -0.2139549935705085E+00
| WP = -0.7784019977592926E-01 | WDP 0.2991532418202351E-01
| WP = -0.5853738010153663E-01 | WDP 0.3726883883267810E+00
| WP = 0.1793442757688947E-04 | WDP 0.8150644679568988E+00

END OF SEARCHING IN THE FOLLOWING LIMITS
-0.700000000000000 0.500000000000000 No.OF SOLUTIOS 1
^^

Input the values of P(0) & ALFA & BETA

P(0) = 50.0000000000000 ALFA = 10.0000000000000 BETA =
10.0000000000000
^^

SOLUTION No. 1
WNEW & WPNEW (at x= 1.0D0 ,A(2),A(3)) =
0.292240929622359D-002 -0.421476760860690D-002 2.06499999999999
-4.12500000000001
****
THE COEFFICIENTS OF POWER SERIES SOLUTION & NLT AA(N)ARE
****

A(0) =	0.0000000000000000E+00	AA(0) =	0.0000000000000000E+00
A(1) =	0.0000000000000000E+00	AA(1) =	0.0000000000000000E+00
A(2) =	0.2064999999999988E+01	AA(2) =	0.0000000000000000E+00
A(3) =	-0.4125000000000008E+01	AA(3) =	0.0000000000000000E+00
A(4) =	0.2083333333333333E+01	AA(4) =	0.0000000000000000E+00
A(5) =	0.0000000000000000E+00	AA(5) =	0.0000000000000000E+00
A(6) =	-0.5736111111111077E-01	AA(6) =	0.8805624624999846E+01
A(7) =	0.4910714285714295E-01	AA(7) =	-0.5276978437499949E+02
A(8) =	-0.1240079365079365E-01	AA(8) =	0.1320632031249995E+03
A(9) =	0.0000000000000000E+00	AA(9) =	-0.1766660156250000E+03
A(10) =	-0.1735766570216019E-01	AA(10) =	0.1325018750312503E+03
A(11) =	0.6656651165674539E-01	AA(11) =	-0.5015107109375015E+02
A(12) =	-0.1111538740162868E+00	AA(12) =	0.1965062210648179E+01
A(13) =	0.1029522235576923E+00	AA(13) =	0.7365815662202375E+01
A(14) =	-0.5514673550014491E-01	AA(14) =	-0.4433761356418727E+01
A(15) =	0.1528831031199433E-01	AA(15) =	0.2941906114444876E+01
A(16) =	-0.4244295642472281E-03	AA(16) =	-0.6205620829689921E+01
A(17) =	-0.1307557402969199E-02	AA(17) =	0.1291903169711088E+02
A(18) =	0.6112347619715240E-03	AA(18) =	-0.1811666250534811E+02
A(19) =	-0.3178958574945036E-03	AA(19) =	0.1743348274386794E+02
A(20) =	0.5337156225708779E-03	AA(20) =	-0.1153049411673100E+02

****

A(1) =	0.0000000000000000E+00	AA(1) =	0.0000000000000000E+00
A(2) =	0.4349999999999971E+01	AA(2) =	0.0000000000000000E+00
A(3) =	-0.8480000000000011E+01	AA(3) =	0.0000000000000000E+00
A(4) =	0.4166666666666667E+01	AA(4) =	0.0000000000000000E+00
A(5) =	0.0000000000000000E+00	AA(5) =	0.0000000000000000E+00
A(6) =	-0.1208333333333325E+00	AA(6) =	0.8231287499999837E+02
A(7) =	0.1009523809523811E+00	AA(7) =	-0.4813883999999942E+03
A(8) =	-0.2480158730158730E-01	AA(8) =	0.1174961969999993E+04
A(9) =	0.0000000000000000E+00	AA(9) =	-0.1532000191999997E+04
A(10) =	-0.1630794477513195E+00	AA(10) =	0.1118583093750001E+04
A(11) =	0.6076861712361640E+00	AA(11) =	-0.4091920523809533E+03
A(12) =	-0.9890043505157335E+00	AA(12) =	0.9378305284391926E+01
A(13) =	0.8927740046620032E+00	AA(13) =	0.6386310209523797E+02
A(14) =	-0.4655427964961079E+00	AA(14) =	-0.4481002983692927E+02
A(15) =	0.1247205025060187E+00	AA(15) =	0.8041692551533013E+02
A(16) =	-0.1920627503176784E-02	AA(16) =	-0.2438348541399445E+03
A(17) =	-0.1133681304269958E-01	AA(17) =	0.5005484262852282E+03
A(18) =	0.6164974487122192E-02	AA(18) =	-0.6814098127042361E+03
A(19) =	-0.8658157681655933E-02	AA(19) =	0.6377027363450534E+03
A(20) =	0.2096979487164153E-01	AA(20) =	-0.4096567333094877E+03

****

THE VALUES OF DEFLECTIONS ALONG THE BEAM ARE

****

X = 0.0000E+00	W = 0.0000000000000000E+00
X = 0.1000E+00	W = 0.3543655566941992E-01
X = 0.2000E+00	W = 0.1128201543860070E+00
X = 0.3000E+00	W = 0.1962220733251517E+00
X = 0.4000E+00	W = 0.2595975482765900E+00
X = 0.5000E+00	W = 0.2867007475533264E+00
X = 0.6000E+00	W = 0.2710158426553360E+00
X = 0.7000E+00	W = 0.2157339484504277E+00
X = 0.8000E+00	W = 0.1338795240201344E+00
X = 0.9000E+00	W = 0.4940256607460826E-01
X = 0.1000E+01	W = 0.4757381756344636E-02

****

THE VALUES OF SLOPE AND 2ND DERV.W' ALONG THE BEAM ARE

****

WP = 0.0000000000000000E+00	WDP 0.8699999999999942E+01
WP = 0.6322601024220101E+00	WDP 0.4111678419440776E+01
WP = 0.8555436692942496E+00	WDP 0.5194537208640521E+00
WP = 0.7691015830627954E+00	WDP -0.2084284118580228E+01
WP = 0.4713436384105593E+00	WDP -0.3708263898340021E+01
WP = 0.5986566812530606E-01	WDP -0.4359597709189453E+01
WP = -0.3682564453374209E+00	WDP -0.4041068129809329E+01
WP = -0.7157776042446525E+00	WDP -0.2743995755586882E+01
WP = -0.8821817732131508E+00	WDP -0.3896368895607834E+00
WP = -0.7416006468092939E+00	WDP 0.3599270134529222E+01
WP = -0.2703137459988483E-02	WDP 0.1275047724141837E+02

THE VALUES OF DEFLECTIONS ALONG THE BEAM ARE
****
X = 0.0000E+00 | W = 0.0000000000000000E+00
X = 0.1000E+00 | W = 0.1673328075775695E-01
X = 0.2000E+00 | W = 0.5293025825398927E-01
X = 0.3000E+00 | W = 0.9131808021197432E-01
X = 0.4000E+00 | W = 0.1195703783622805E+00
X = 0.5000E+00 | W = 0.1302703439482897E+00
X = 0.6000E+00 | W = 0.1208829053499005E+00
X = 0.7000E+00 | W = 0.9374423769458129E-01
X = 0.8000E+00 | W = 0.5607386880882263E-01
X = 0.9000E+00 | W = 0.2002748190319678E-01
X = 0.1000E+01 | W = 0.2922409296223560E-02
****
THE VALUES OF SLOPE AND 2ND DERV.W' ALONG THE BEAM ARE
****
| WP = 0.0000000000000000E+00 | WDP 0.4129999999999976E+01
| WP = 0.2975802253836006E+00 | WDP 0.1904847837606268E+01
| WP = 0.3975772271709029E+00 | WDP 0.1778607600097873E+00
| WP = 0.3496417156651223E+00 | WDP -0.1054454514167737E+01
| WP = 0.2030477330013194E+00 | WDP -0.1795894370582559E+01
| WP = 0.6728302748736997E-02 | WDP -0.2049309651507864E+01
| WP = -0.1905852798918143E+00 | WDP -0.1815783996194793E+01
| WP = -0.3401625239457745E+00 | WDP -0.1094185364691951E+01
| WP = -0.3930112445853027E+00 | WDP 0.1200588587610390E+00
| WP = -0.2991860718751643E+00 | WDP 0.1845039544732279E+01
| WP = -0.4214767608607760E-02 | WDP 0.4173034082062462E+01

END OF SEARCHING IN THE FOLLOWING LIMITS
-3.50000000000000 2.50000000000000 No.OF SOLUTIOS 1

^^

Input the values of P(0) & ALFA & BETA

P(0) = 100.000000000000 ALFA = 10.0000000000000 BETA =
10.0000000000000
^^

SOLUTION No. 1

WNEW & WPNEW (at x= 1.0D0 ,A(2),A(3)) =
0.475738175634464D-002 -0.270313745998571D-002 4.34999999999997
-8.48000000000001

****
THE COEFFICIENTS OF POWER SERIES SOLUTION & NLT AA(N)ARE
****
A(0) = 0.0000000000000000E+00 | AA(0) = 0.0000000000000000E+00

END OF SEARCHING IN THE FOLLOWING LIMITS
-7.00000000000000 5.00000000000000 No.OF SOLUTIOS 1

^^

Input the values of P(0) & ALFA & BETA

P(0) = 200.000000000000 ALFA = 10.0000000000000 BETA =
10.0000000000000

^^

NO SOLUTION WITHIN THE GIVEN RANGE WITH THE FOLLOWING
PARAMETERS 200.000000000000 10.0000000000000
10.0000000000000
END OF SEARCHING IN THE FOLLOWING LIMITS
-14.0000000000000 10.0000000000000 No.OF SOLUTIOS 0

ملخص باللغة العربية

حل انحراف الجيزان على ارضية غير خطية بطريقة المتسلسلات الاسية

اعداد

محمد محمود سلامة الحسبان

اشراف

الدكتور مازن القيسي

انحرافات الجيزان على ارضية غير خطية تحت تأثير قوى ساكنة تم دراستها وتحليلها بعدة طرق تقريبية، وفي هذه الدراسة ولاول مرة سيتم حل هذه الانحرافات بطريقة المتسلسلات الاسية. حيث ان المعادلة التي تحكم حركة الجيزان هي معادلة تفاضلية من الدرجة الرابعة، وتعتمد على عدة عوامل وهي: معامل المرونة الخطي للارضيةو معامل المرونة غير الخطي للارضية و نوع مادة الجيزان و شكل مقطع الجيزان ونوع القوى المؤثرة عاموديا على الجيزان. لقد تم افتراض الانحراف على شكل متسلسلة اسية لنوعين من الجيزان اولهما مثبت من كلا الطرفين والاخر مثبت جزئيا من كلا الطرفين تحت تأثير ثلاث قوى مختلفة الشكل وهي : قوة ثابتة وقوة غير متماثلة من الدرجة الثالثة واخيرا على شكل موجة جيبية. بداية حل الانحراف اخذ من حل المعادلة التفاضلية بدون وجود ارضية وهو ما يسمى الحل الخطي، ثم بدأ ايجاد معاملات المتسلسلة الاسية عن طريق برنامج كمبيوتر بلغة (فورتران) وذلك بتحقيق الشروط عند اطراف الجيزان على ارضية غير خطية.

تم دراسة تأثير العوامل المؤثرة في المعادلة على انحراف الجيزان لكل حالة على حده، بالاضافة الى دراسة الضغط الناتج عن تلك القوى، وتم رسم الاشكال الخاصة بذلك، كما تم دراسة تأثير درجة المتسلسلة الاسية على الانحرافات عند وسط الجيزان، و تم مقارنة الحل بهذه الطريقة مع طريقة (الاحساس المتوازن)، فوجد ان طريقة المتسلسلات الاسية طريقة فعالة جدا اذا تمت برمجتها.